

# Atomic Pair Distribution Function (PDF) Analysis

**Katharine Page**

Diffraction Instrument Scientist

Neutron Scattering Division

Oak Ridge National Laboratory

ORNL is managed by UT-Battelle, LLC for the US Department of Energy

# Atomic Pair Distribution Function (PDF) Analysis

*Starting August 1*

Assistant Professor

Materials Science and Engineering Department

University of Tennessee, Knoxville

THE UNIVERSITY of  
**TENNESSEE** **UT**  
KNOXVILLE

**JIAM**   
JOINT INSTITUTE FOR ADVANCED MATERIALS

ORNL is managed by UT-Battelle, LLC for the US Department of Energy

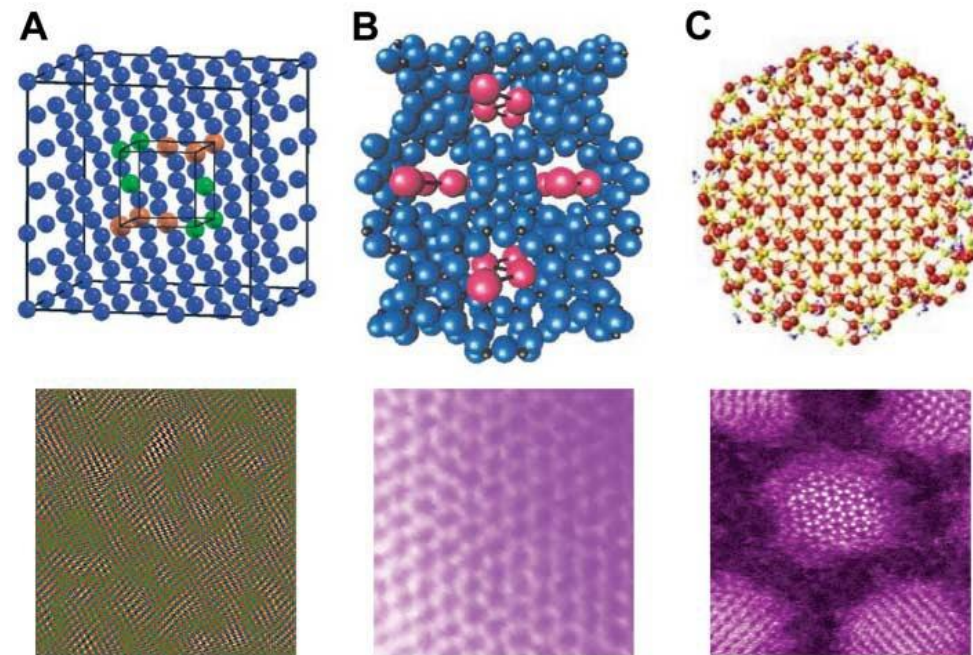
 **Shull Wollan Center**  
a Joint Institute for Neutron Sciences

# Outline

- Local Structure and Total Scattering
- The Pair Distribution Function (PDF)
- Applications
  - Local Distortions
  - Chemical Short-Range Order
  - Nanomaterial Structure
  - Amorphous Structures
- Experimental Considerations
- Modeling a PDF
- Emerging Areas

# What is a *local structure*?

- **Disordered materials:** The interesting properties are often governed by the defects or local structure
- **Non crystalline materials:** Amorphous solids, liquids, glasses and polymers
- **Nanostructures:** Well defined local structure, but long-range order limited to nanometers lengthscales (poorly defined Bragg peaks)

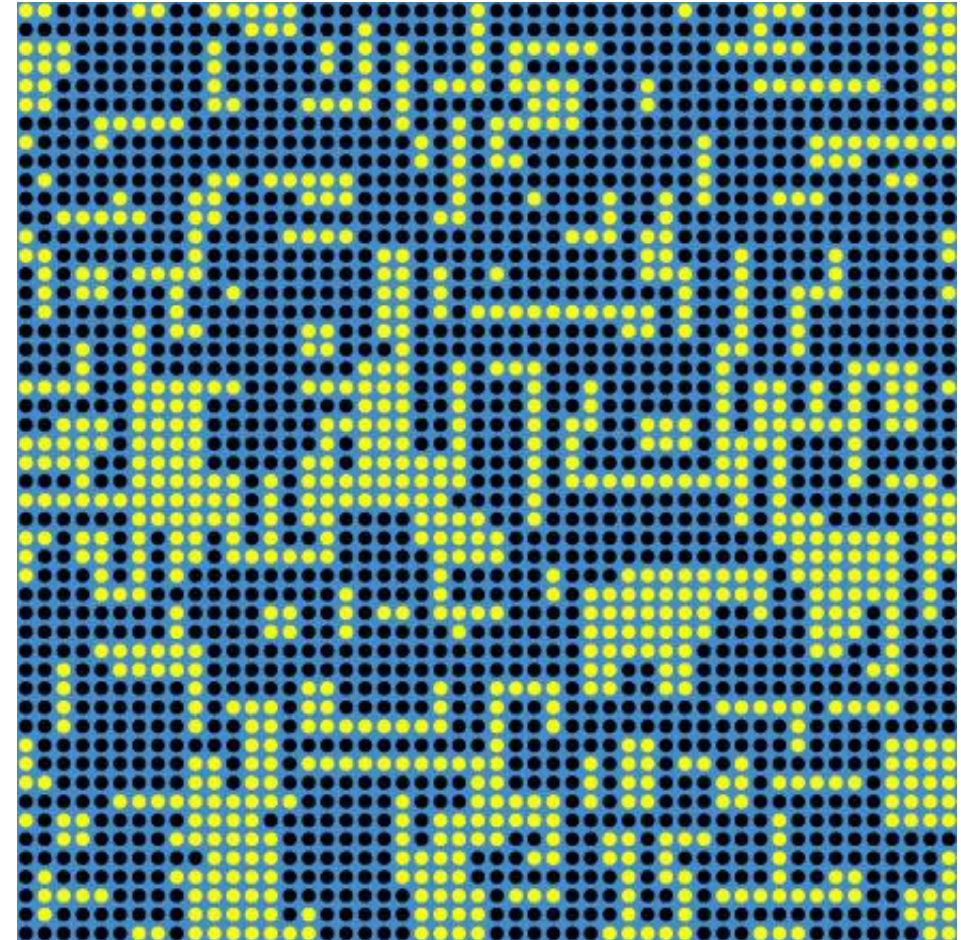
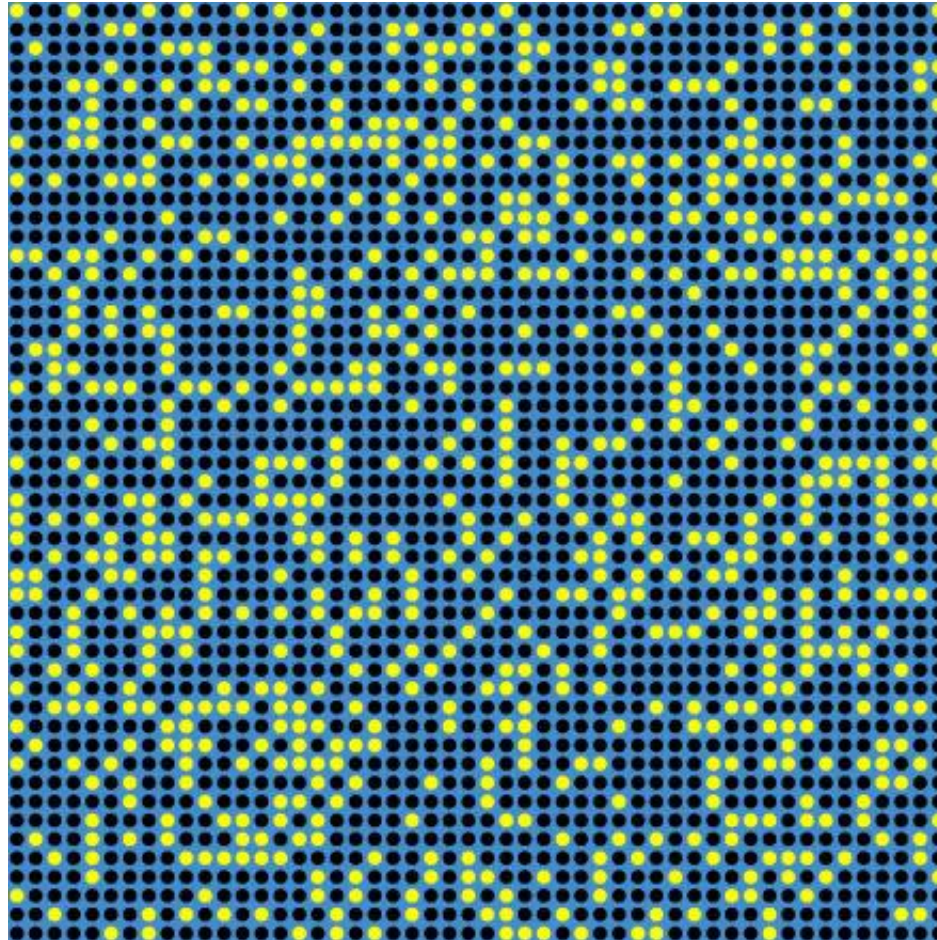


S.J.L. Billinge and I. Levin, **The Problem with Determining Atomic Structure at the Nanoscale**, *Science* **316**, 561 (2007).

D. A. Keen and A. L. Goodwin, **The crystallography of correlated disorder**, *Nature* **521**, 303–309 (2015).

# What is *total* scattering?

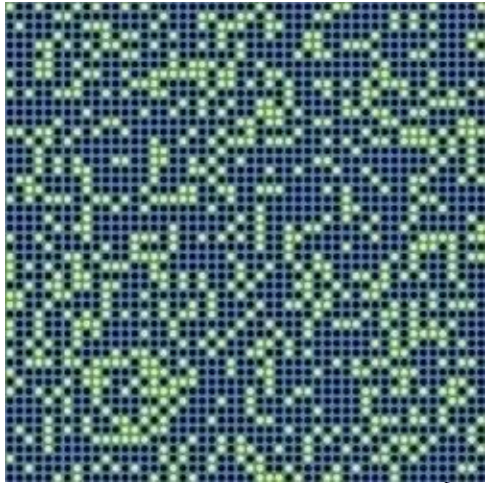
Courtesy of Thomas Proffen



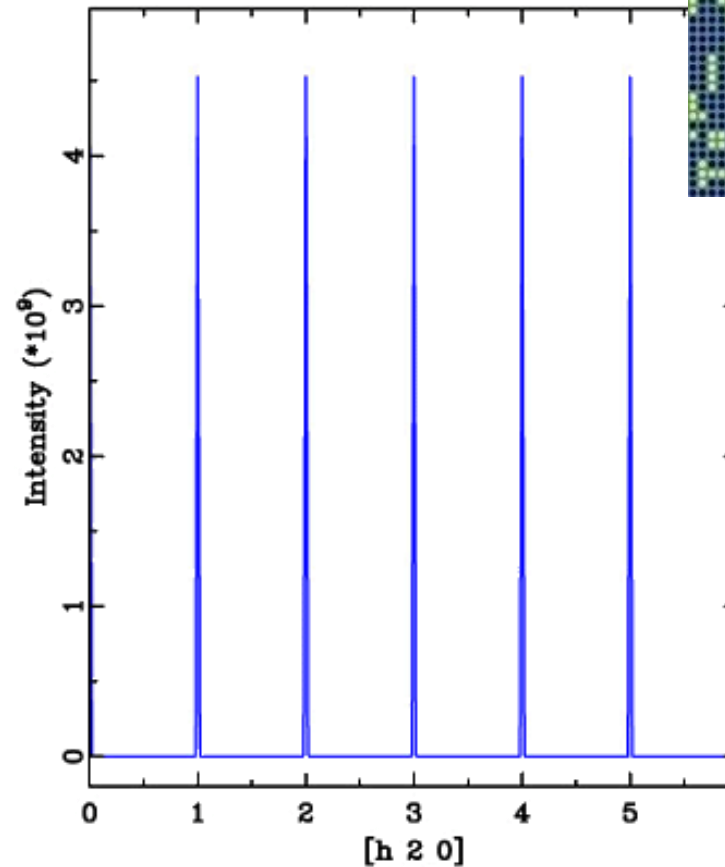
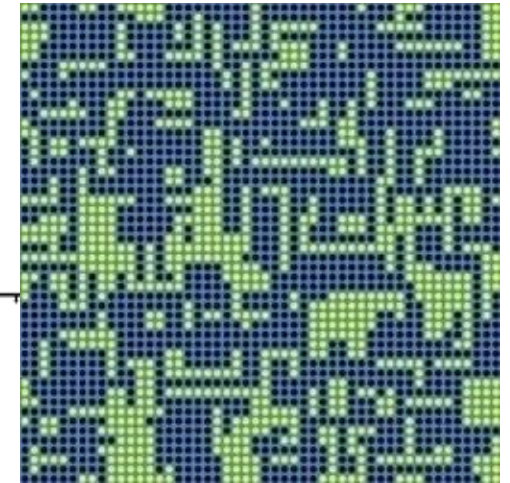
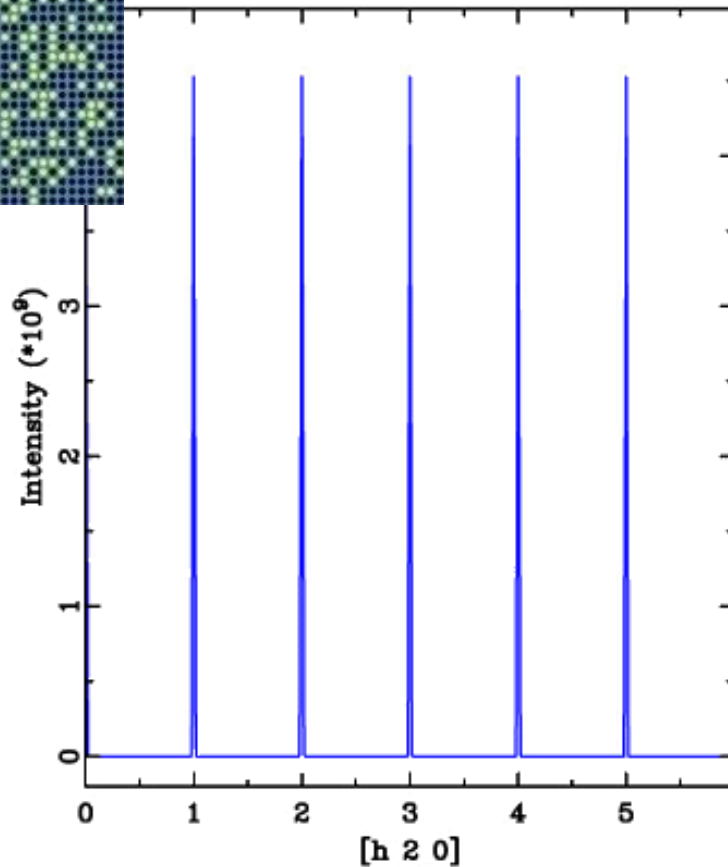
Cross section of 50x50x50 unit cell model crystal consisting of 70% blue atoms and 30% *vacancies*.

**Properties might depend on vacancy ordering!**

# Bragg peaks are “blind”

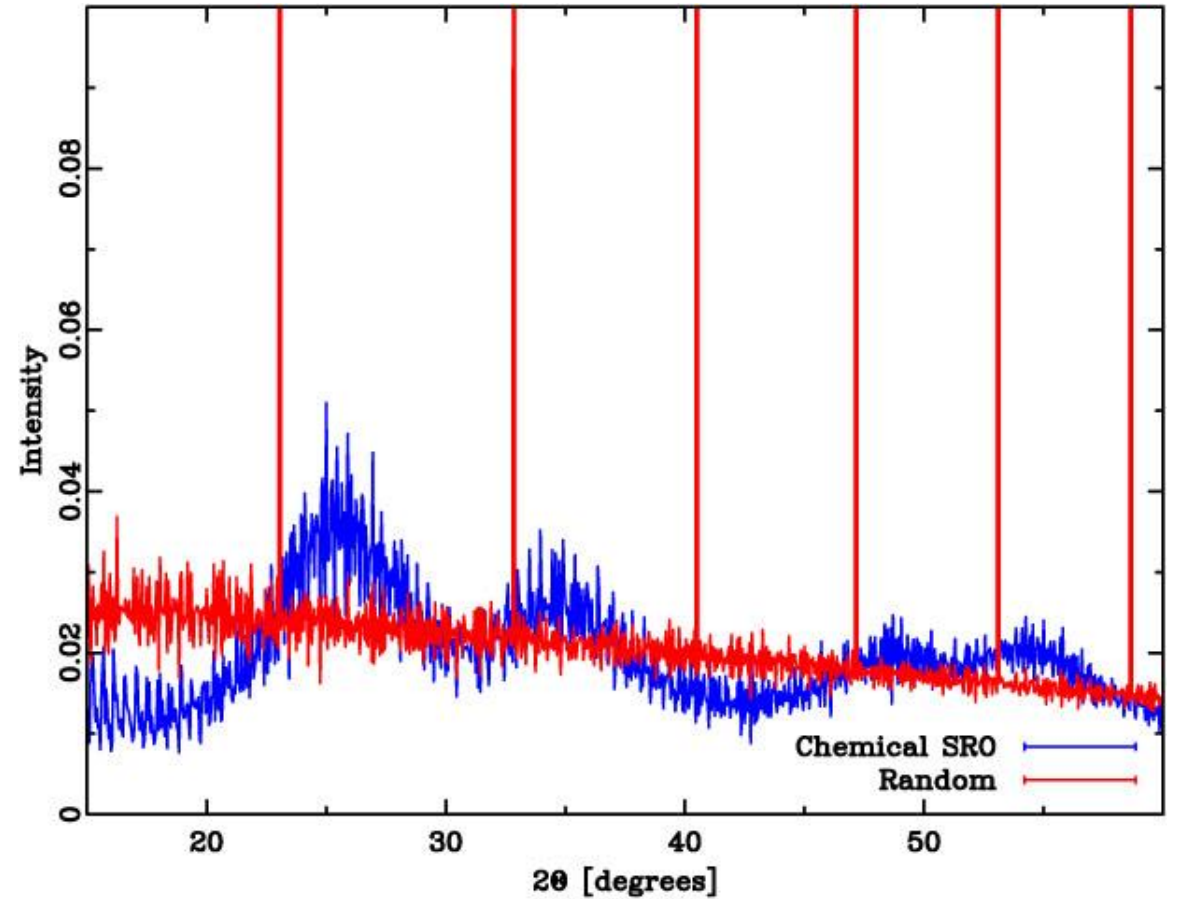
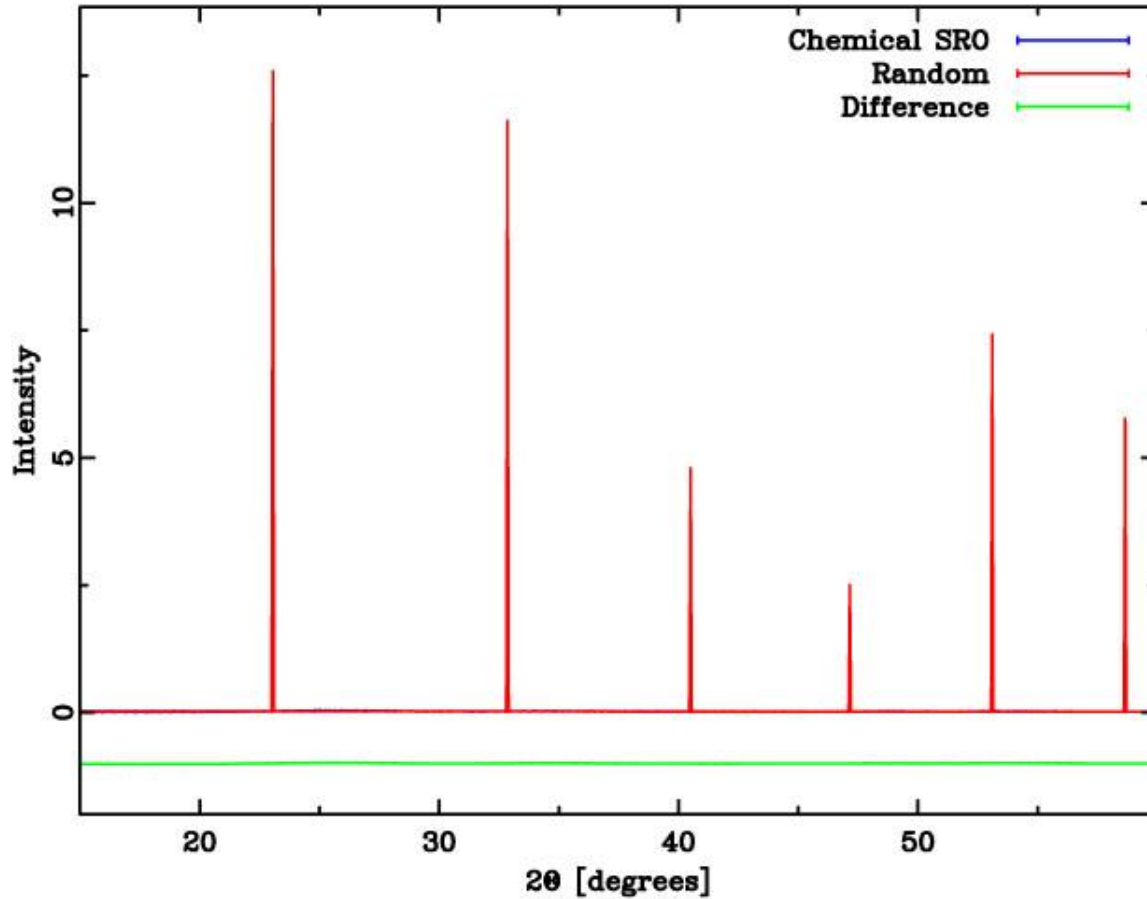


**Bragg scattering:** Information about the *average* structure, e.g. average positions, displacement parameters and occupancies.



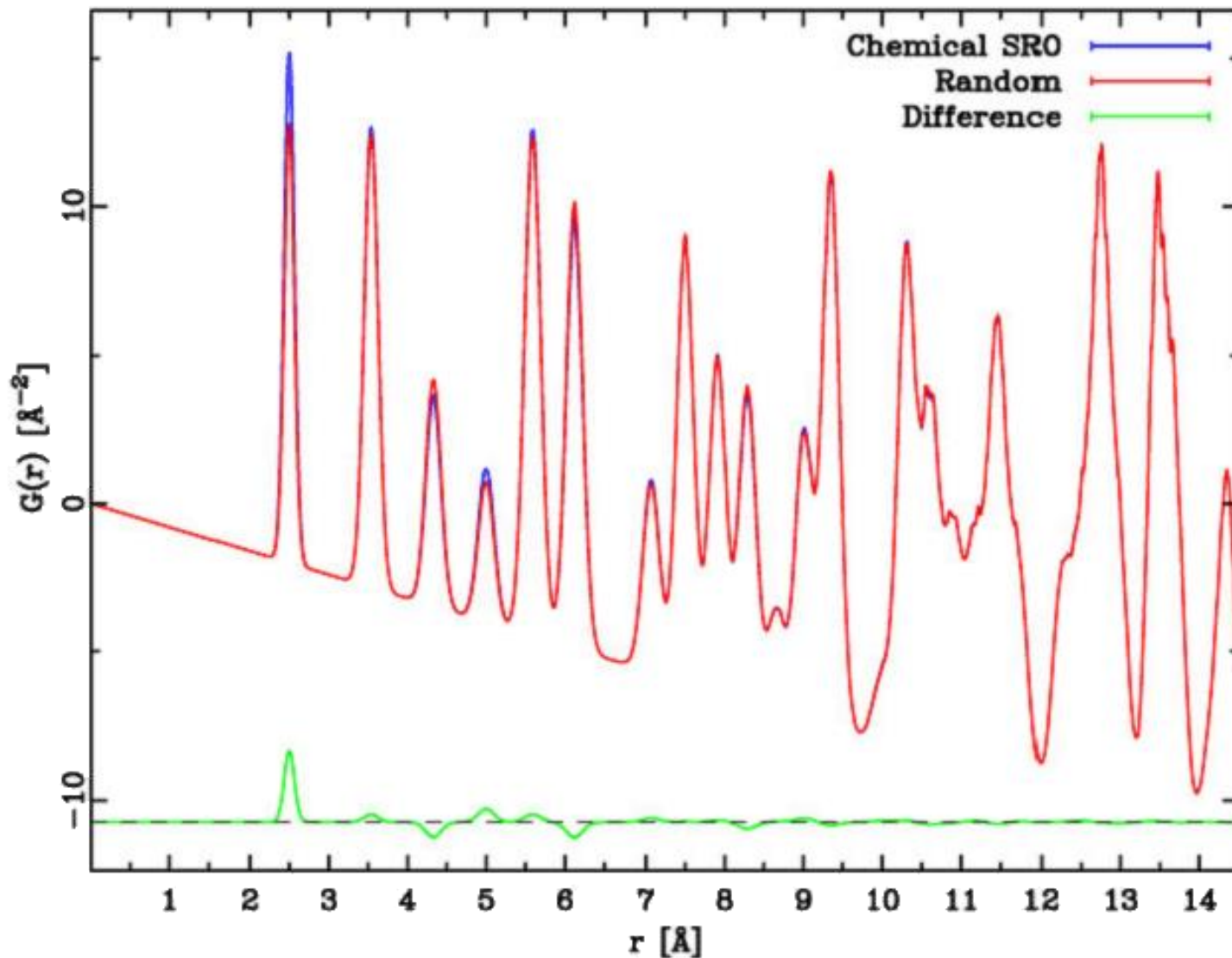
Courtesy of Thomas Proffen

# How about powder diffraction?



Courtesy of Thomas Proffen

# The Pair Distribution Function

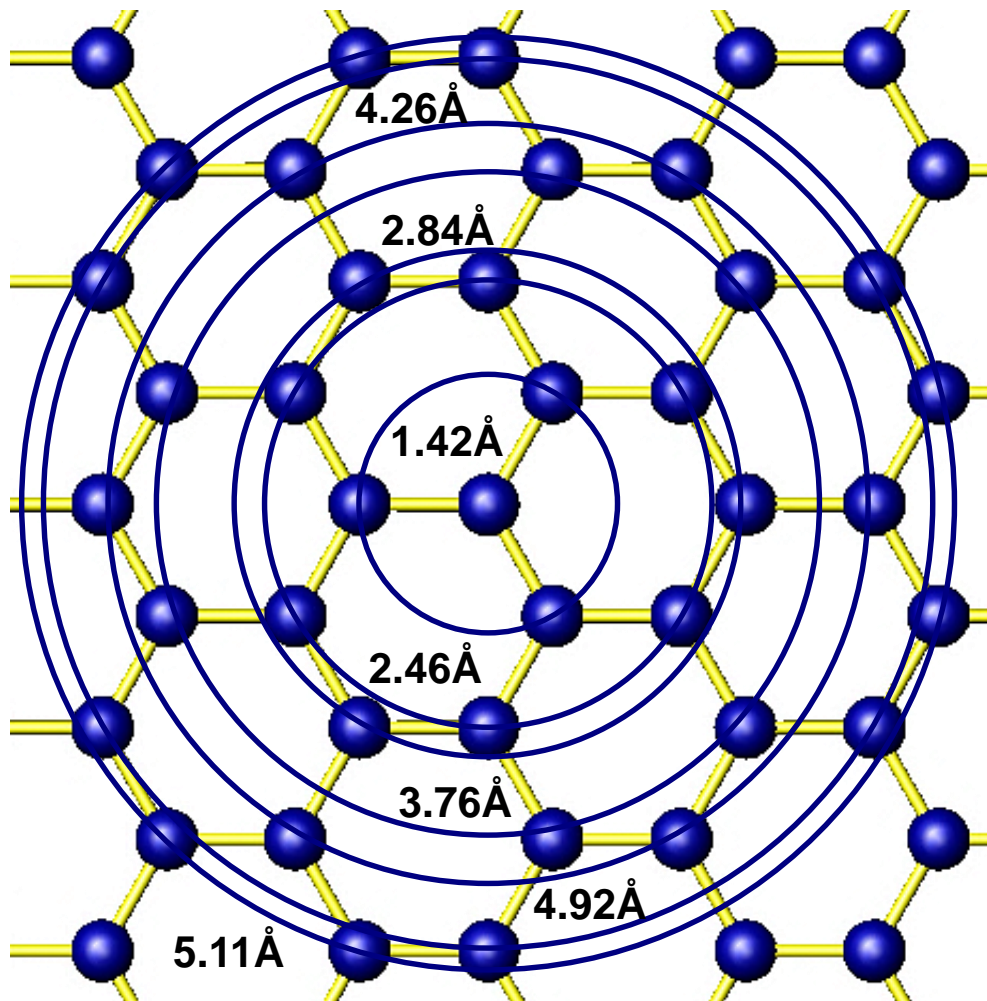


The PDF is the **Sine-Fourier transform** of the **total scattering** (Bragg and diffuse) diffraction pattern

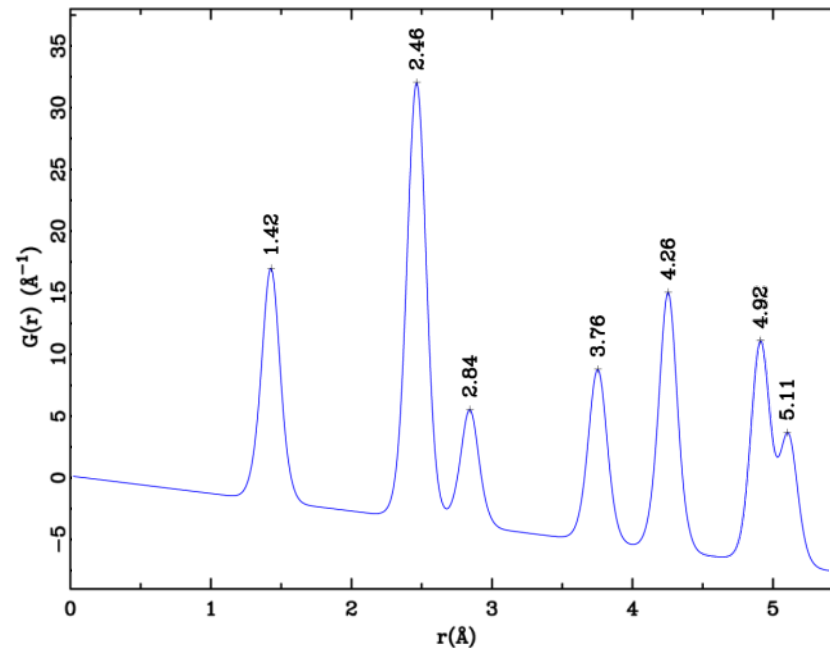
Th. Proffen, Analysis of occupational and displacive disorder using the atomic pair distribution function: a systematic investigation, *Z. Krist.* **215**, 661 (2000).



# What is a PDF?



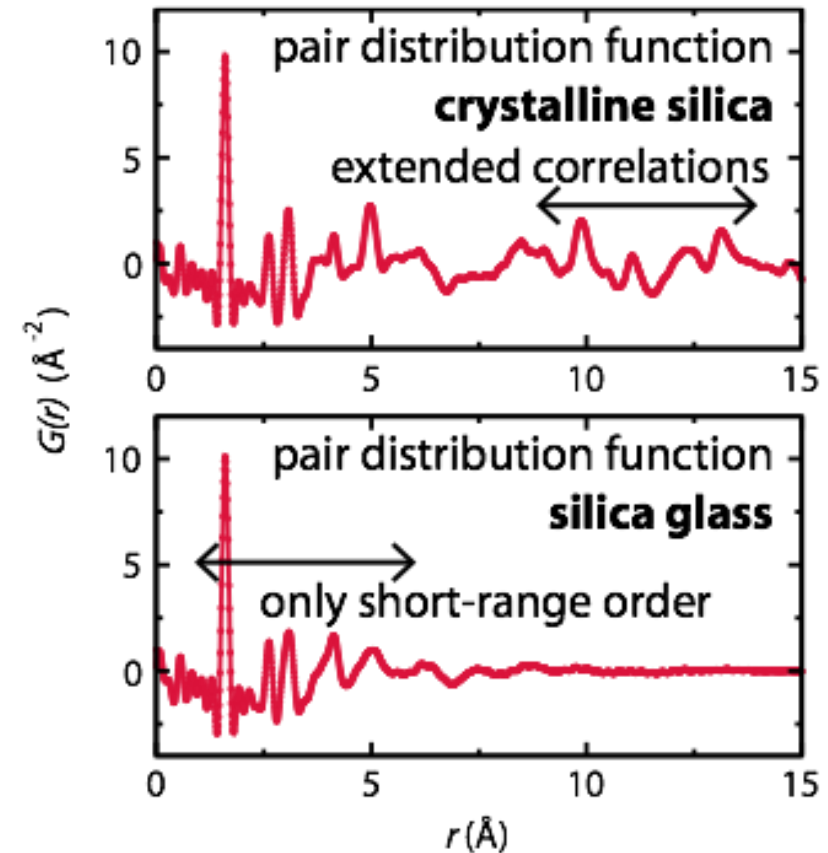
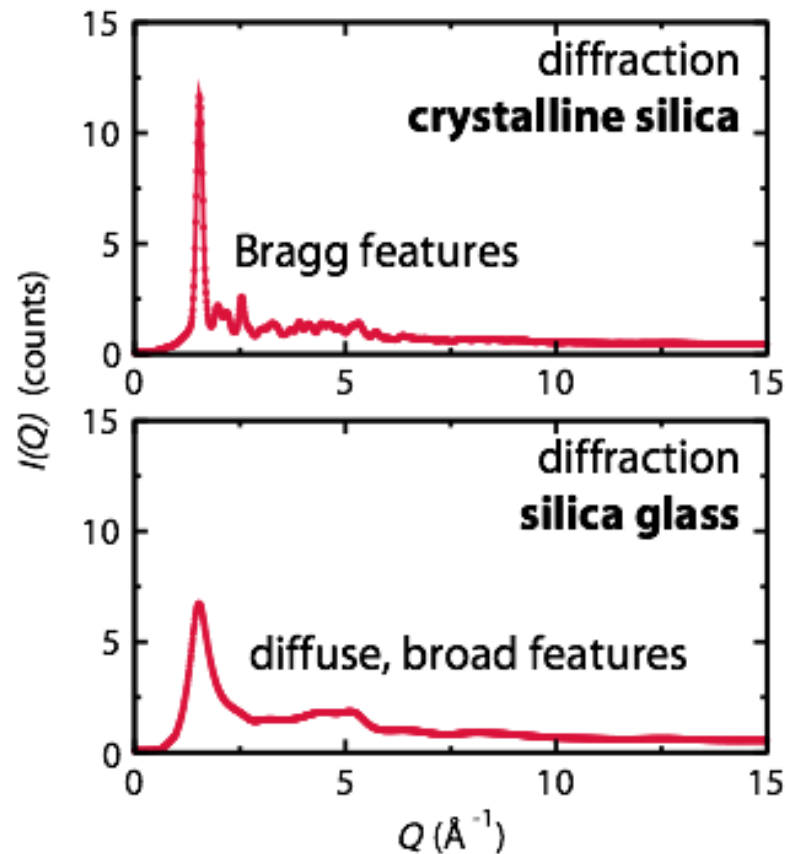
The **Pair Distribution Function (PDF)** gives the probability of finding an atom at a distance “ $r$ ” from a given atom.



# Pair Distribution Function

Sine-Fourier transform of **all** scattered neutron/X-ray intensity (crystalline and amorphous)

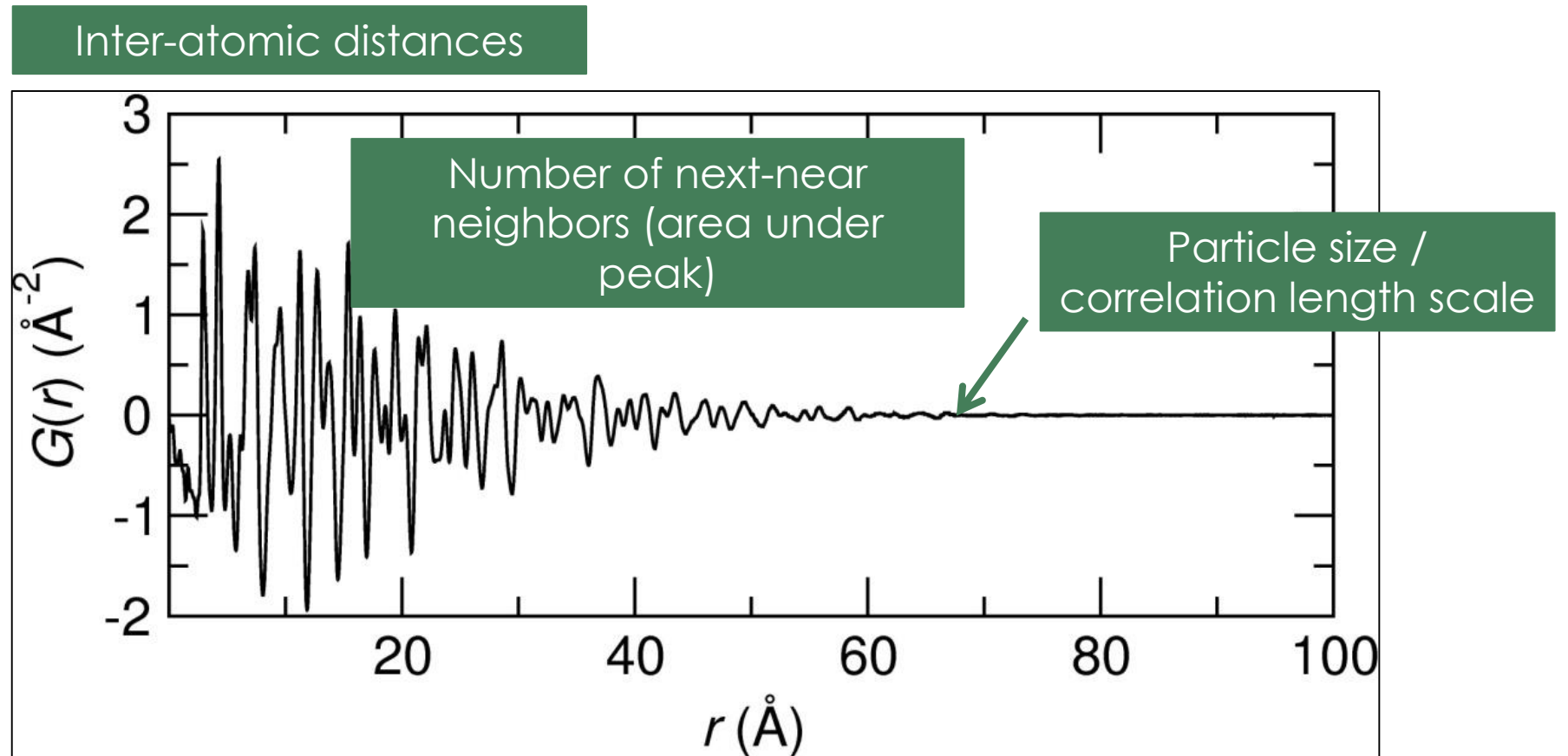
→ *Experimental, ensemble, real-space, atom-atom histogram*



# Pair Distribution Function

## PDF analysis →

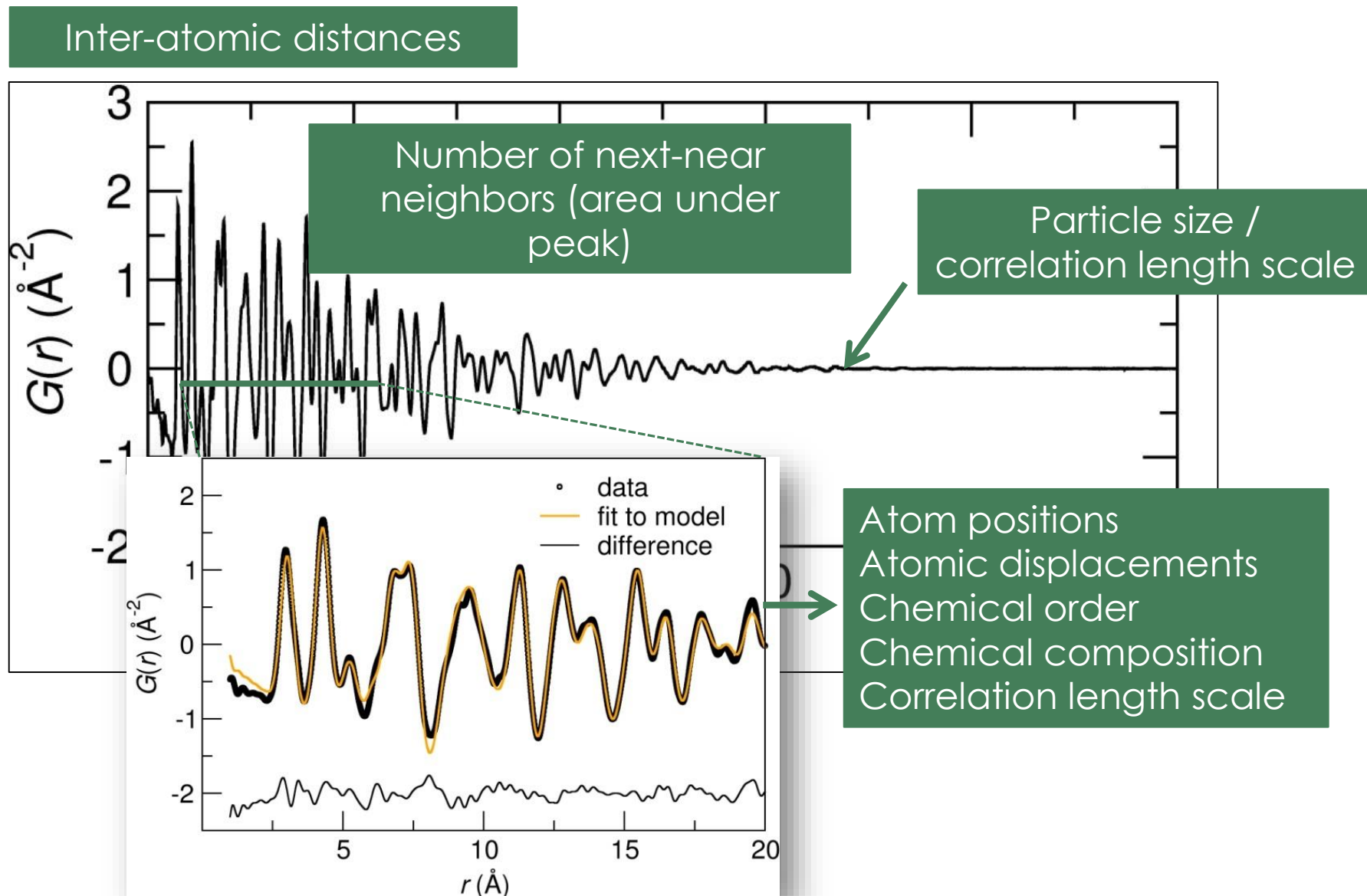
Local atomic structure for disordered crystalline materials, nanomaterials, and amorphous materials



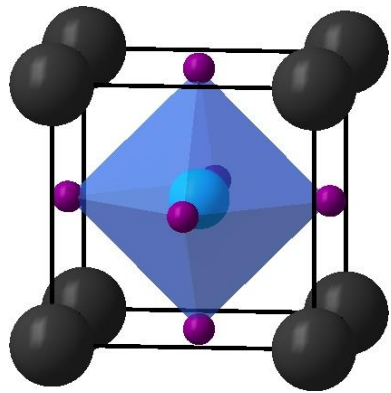
# Pair Distribution Function

## Quantitative analysis →

fitting a model to the data over specific ranges

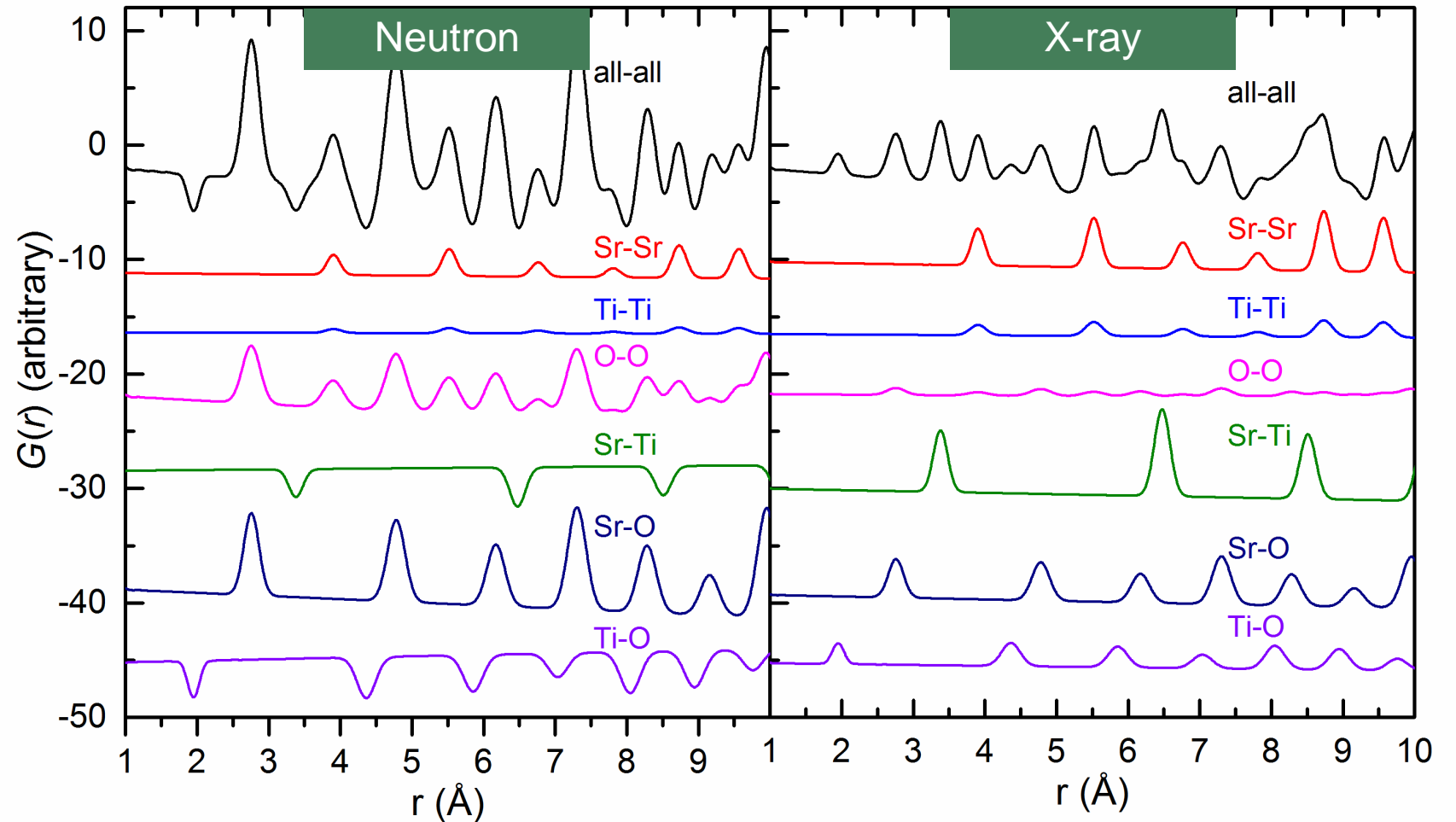


# Partial PDFs



SrTiO<sub>3</sub>

$s(s+1)/2$  partial structure factors characterize a system containing  $s$  species



**Neutron and x-ray PDFs are often highly complementary.**

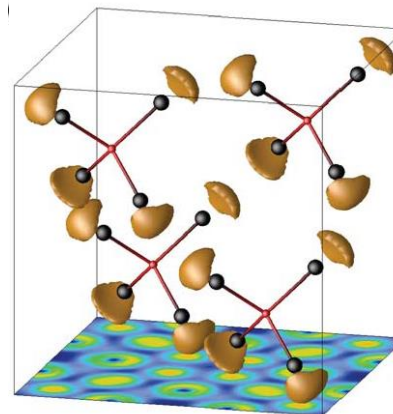
# What types of studies can be done with the PDF technique?

- Local Distortions
- Chemical Short Range Ordering
- Nanostructures
- Amorphous Structures

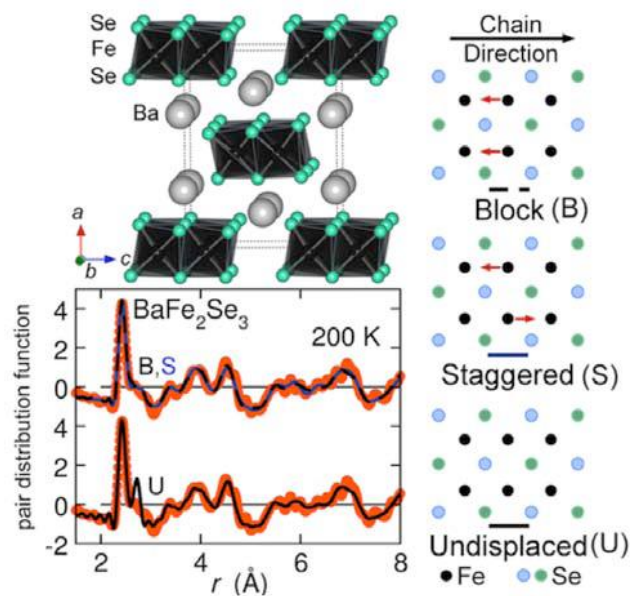


# Local distortions *via* PDF

- Local dipoles
- Local Jahn-Teller distortions
- Frustrated lattices
- Orbital ordering
- *etc.*

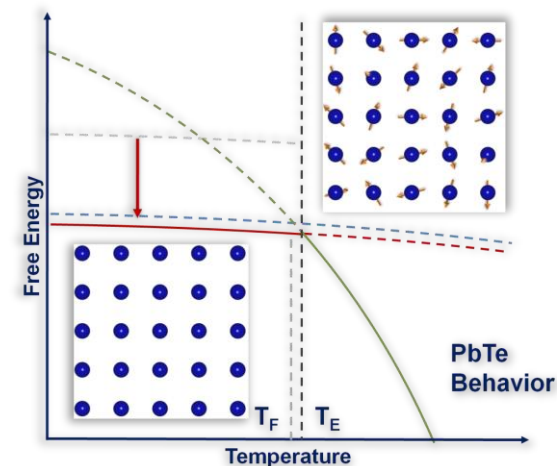


D. P. Shoemaker, *et al.*,  
Reverse Monte Carlo  
neutron scattering study  
of the 'ordered-ice' oxide  
pyrochlore  $\text{Pb}_2\text{Ru}_2\text{O}_{6.5}$ , *J. Phys.: Condens. Matter*  
**23** (2011).



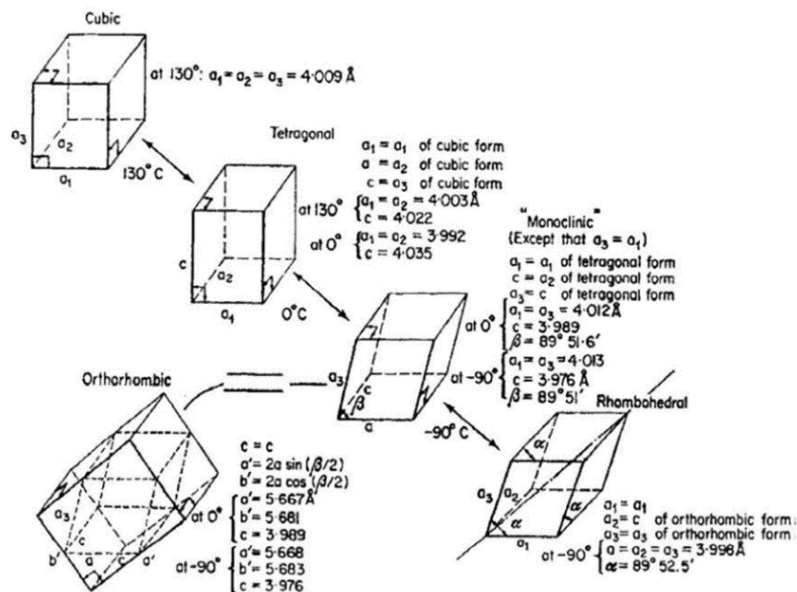
D. Louca, *et al.*,  
Suppression of  
superconductivity in Fe  
pnictides by annealing;  
a reverse effect to  
pressure, *Phys. Rev. B* **84**,  
054522 (2011).

E. Bozin, *et al.*,  
Entropically Stabilized  
Local Dipole  
Formation in Lead  
Chalcogenides,  
*Science* **330**, 1660  
(2010).



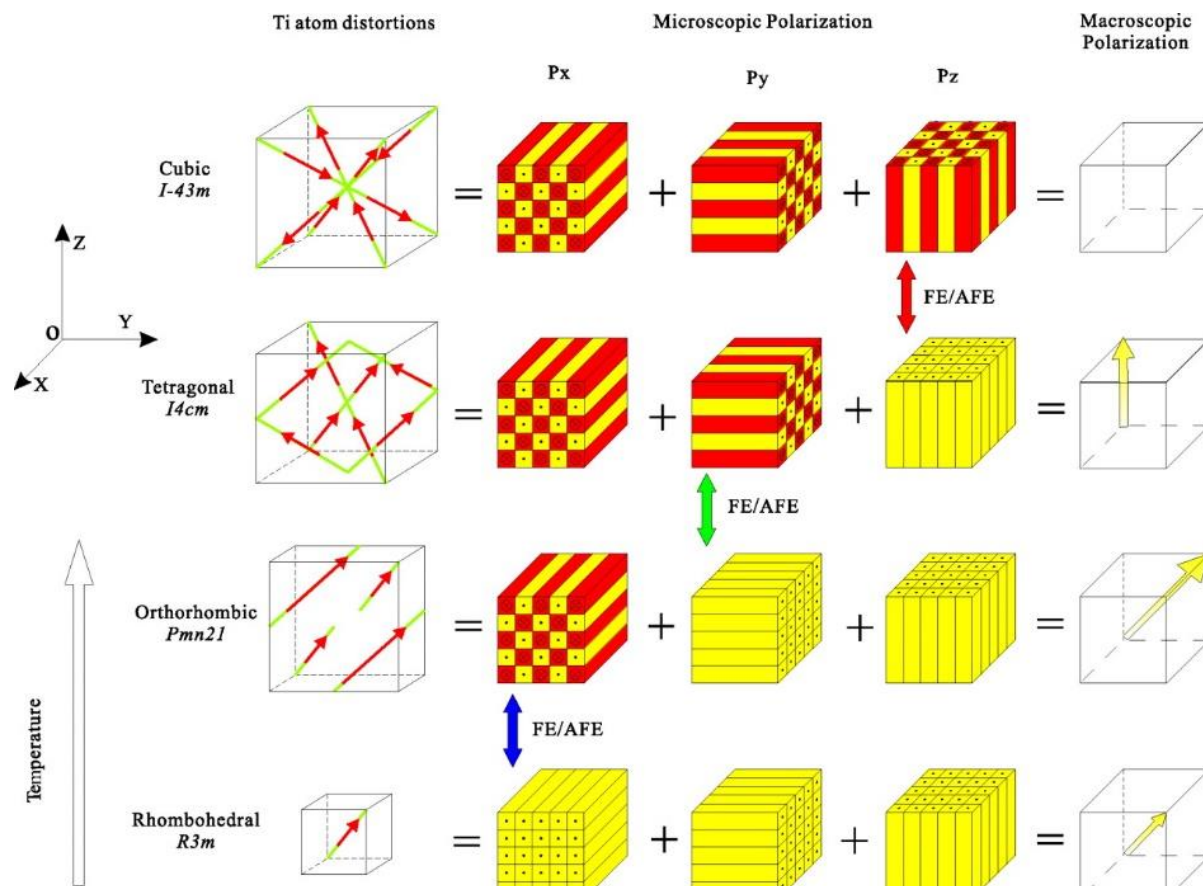
# Example: Local structure in BaTiO<sub>3</sub>

## Crystallographic Phase Transitions



Jaffe, Cook, and Jaffe, *Piezoelectric ceramics*, Academic Press, 1971.

Long-range: cubic  $\rightarrow$  tetragonal  $\rightarrow$  orthorhombic  $\rightarrow$  rhombohedral



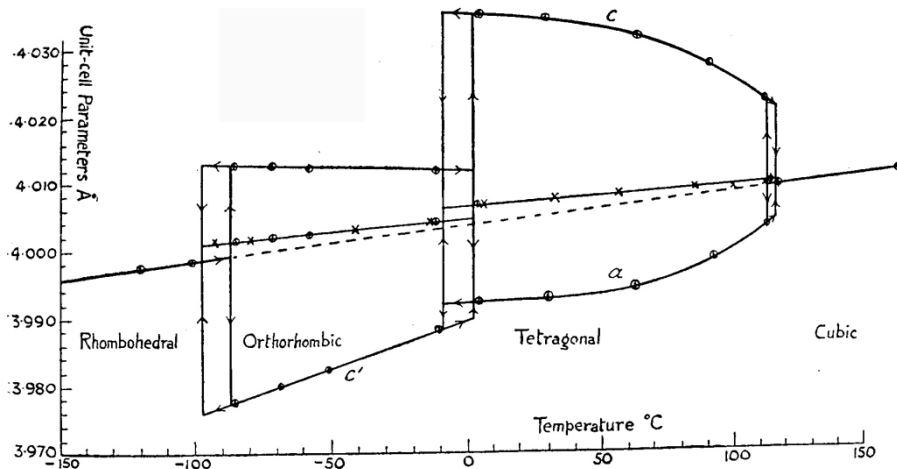
**Locally, Ti<sup>4+</sup> displacements are always along [111] directions (octahedral faces)  $\rightarrow$  Results in 3 short and 3 long Ti-O bonds**

Zhang, Cagin, and Goddard, The ferroelectric and cubic phases in BaTiO<sub>3</sub> ferroelectrics are also antiferroelectric, *PNAS*, **103**, 14695-14700 (2006).

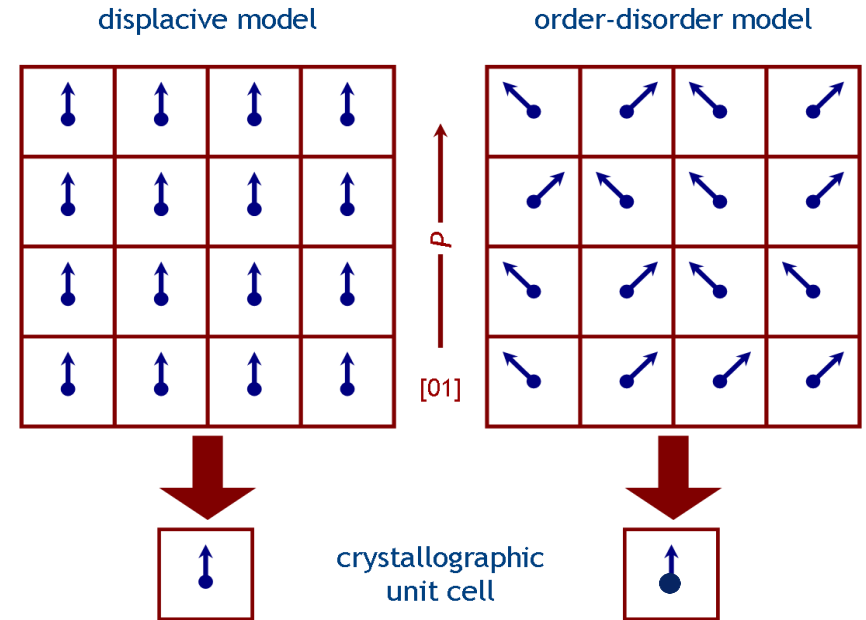


# Example: Local structure in BaTiO<sub>3</sub>

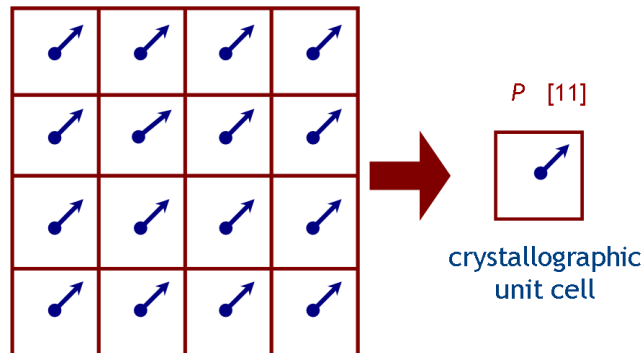
BaTiO<sub>3</sub>: Ferroelectric oxide, a rhombohedral (*R3m*) ground state and a room temperature tetragonal (*P4mm*) structure



2D models with distinct local motifs but similar "average" motifs



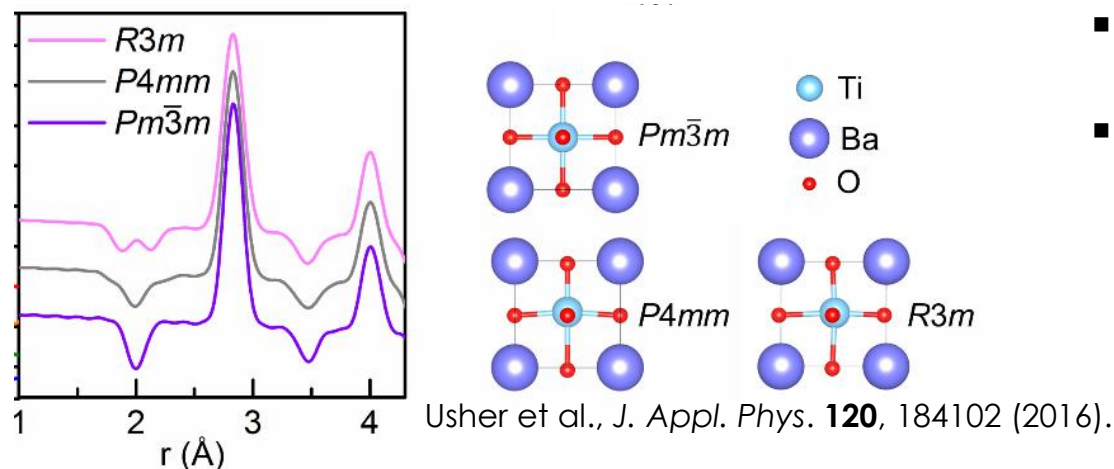
A 2D model where local and average motifs are the same



BaTiO<sub>3</sub> displays order-disorder phenomena: room temperature local structure known to have rhombohedral-like pair-pair correlations

# Neutron PDF for bulk/nano BaTiO<sub>3</sub>

## Calculated BaTiO<sub>3</sub> PDFs

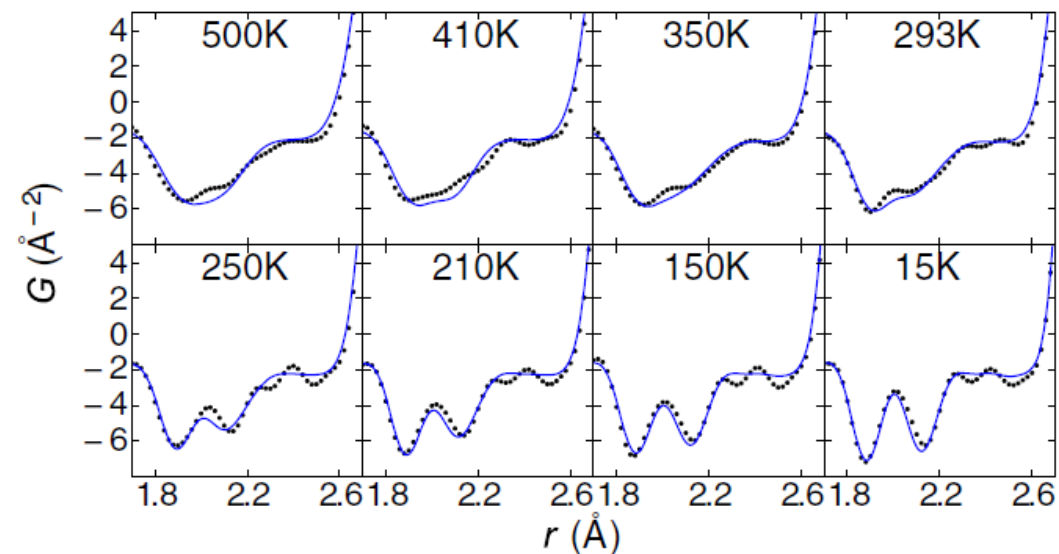


- Neutron PDF is sensitive to Ti-O correlations
- At room temperature, BaTiO<sub>3</sub> locally has a split (*R3m* like) first Ti-O peak, displaying classic order-disorder behavior

## Experimental BaTiO<sub>3</sub> PDFs

M. S. Senn, D. A. Keen, T. C. A. Lucas, J. A. Hriljac, and A. L. Goodwin, **Emergence of Long-Range Order in BaTiO<sub>3</sub> from Local Symmetry-Breaking Distortions**, *Phys. Rev. Lett.* **116**, 207602 (2016).

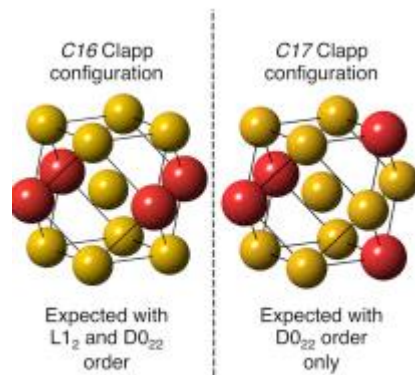
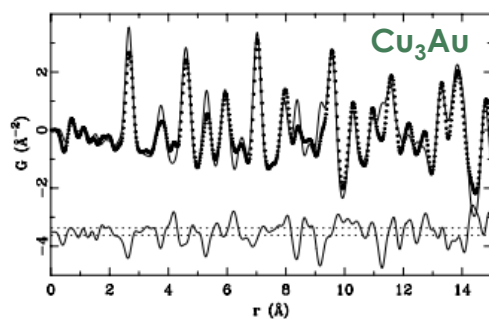
K. Page et al., *Chem. Mater.* **22**, 4386–4391 (2010).  
K. Page, et al., *Phys. Rev. Lett.* **101**, 205502 (2008).



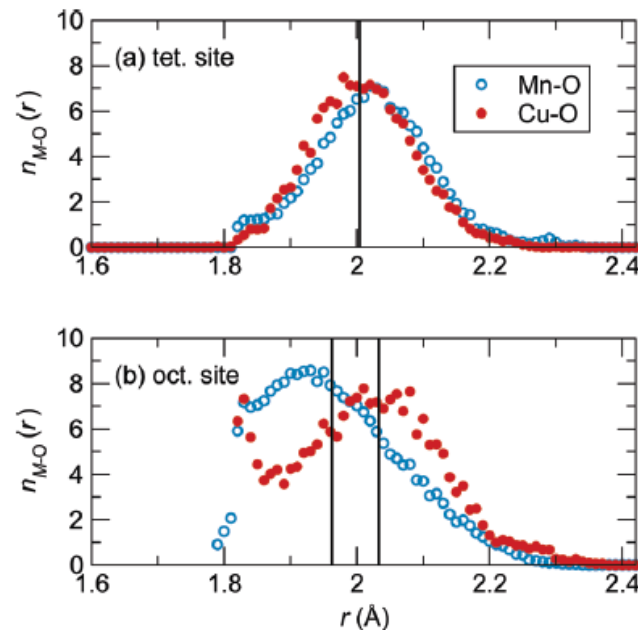
# Chemical Short-Range order via PDF

- Substitution effects
- Chemical clustering
- Ion-specific local environments
- Vacancy ordering

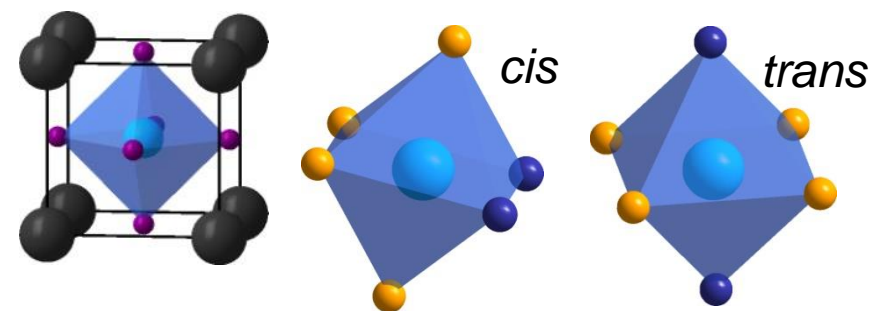
Th. Proffen, V. Petkov, S. J. L. Billinge, and T. Vogt, Chemical short range order obtained from the atomic pair distribution function, *Z. Kristallogr.* **217**, (2002) 47–50.



L.R. Owen, H.Y. Playford, H.J. Stone and M.G. Tucker, Analysis of short-range order in Cu<sub>3</sub>Au using X-ray pair distribution functions. *Acta Materialia* (2017) 125, 15-26.



D. P. Shoemaker, J. Li, and R. Seshadri, Unraveling Atomic Positions in an Oxide Spinel with Two Jahn-Teller Ions: Local Structure Investigation of CuMn<sub>2</sub>O<sub>4</sub>, *J. Am. Chem. Soc.* **131**, 11450 (2009).



K. Page, *et al.*, Local atomic ordering in BaTaO<sub>2</sub>N studied by neutron pair distribution function analysis and density functional theory, *Chem. Mater.* **19** (2007) 4037-4042.

# Example: High Voltage Spinel Cathode ${}^7\text{LiNi}_{0.5}\text{Mn}_{1.5}\text{O}_4$

High operating voltage ( $\sim 4.7$  V versus  $\text{Li}^+/\text{Li}$ ) and facile three-dimensional lithium ionic conductivity  
*Zhong et al., 1997; Ohzuku et al., 1999*

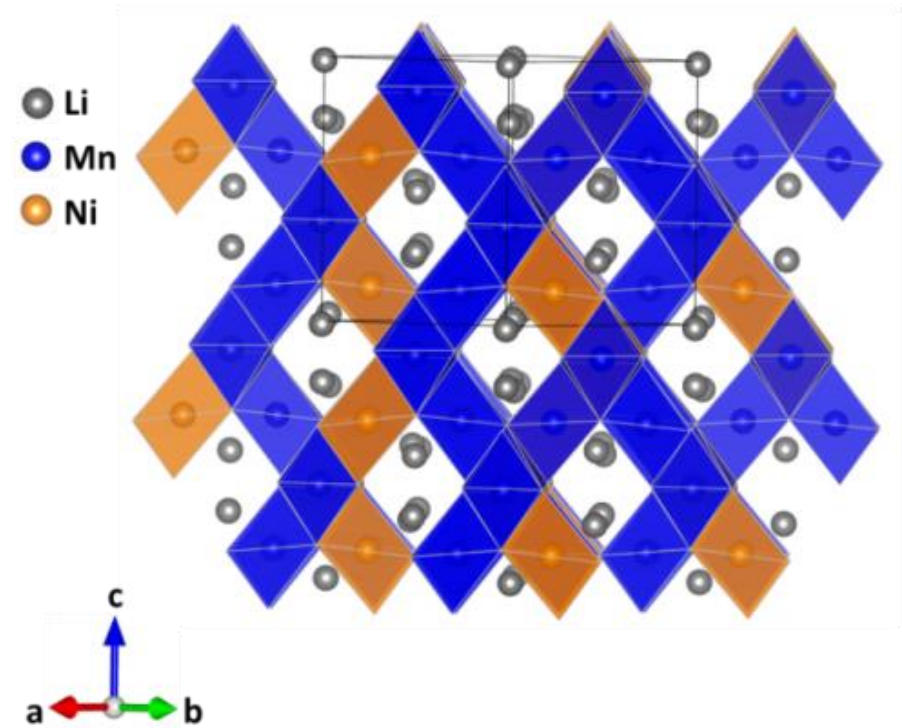
Two distinct polymorphs are known: Ni/Mn cation ordering strongly impacts electrochemical performance  
*Idemoto et al., 2003; Zhong et al., 1997*

(1) Disordered phase (S.G.  $Fd-3m$ ), where Ni/Mn are randomly distributed at the 16d site via high temperature solid state reaction

(2) Long-range cation ordered phase (S.G.  $P4_332$  or  $P4_132$ ) via extended post-annealing at  $700^\circ\text{C}$  to  $600^\circ\text{C}$

*Kunduraci & Amatucci, 2006; Kunduraci et al., 2006; Kim et al., 2004; Ma et al., 2010; Moorhead-Rosenberg et al., 2015*

**We studied the nature and length-scale of local cation ordering in this system and related it to electrochemical performance**



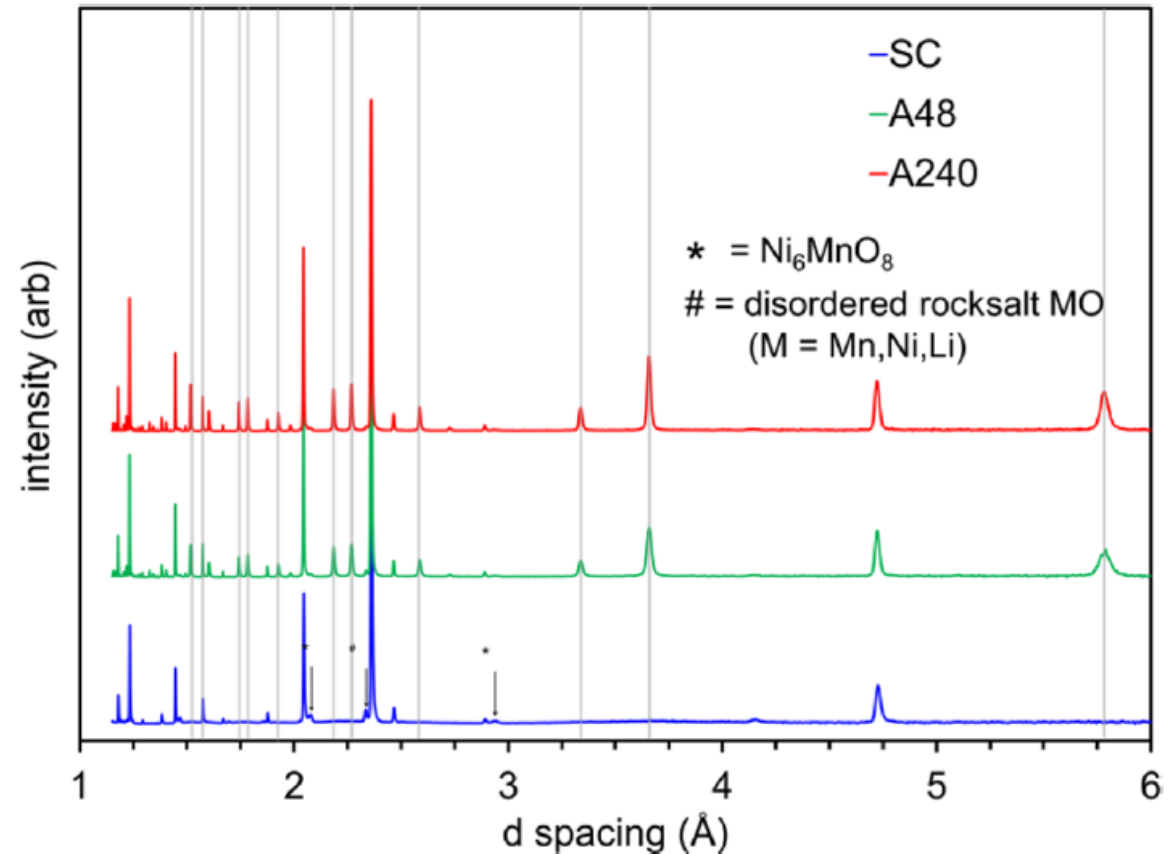
# Example: High Voltage Spinel Cathode ${}^7\text{LiNi}_{0.5}\text{Mn}_{1.5}\text{O}_4$

**Slow Cooled (SC):** 8 hours at 900°C,  
1.5°C/min cooling

**Fast Cooled (FC):** 8 hours at 900°C,  
5°C/min cooling

**Annealed (A48):** 48 hours at 700°C

**Annealed (A240):** 240 hours at 700°C



Cation ordering examined at the POWGEN Beamline, SNS: large nuclear scattering length contrast between nickel ( $b = 10.3$  fm) and manganese ( $b = -3.73$  fm)

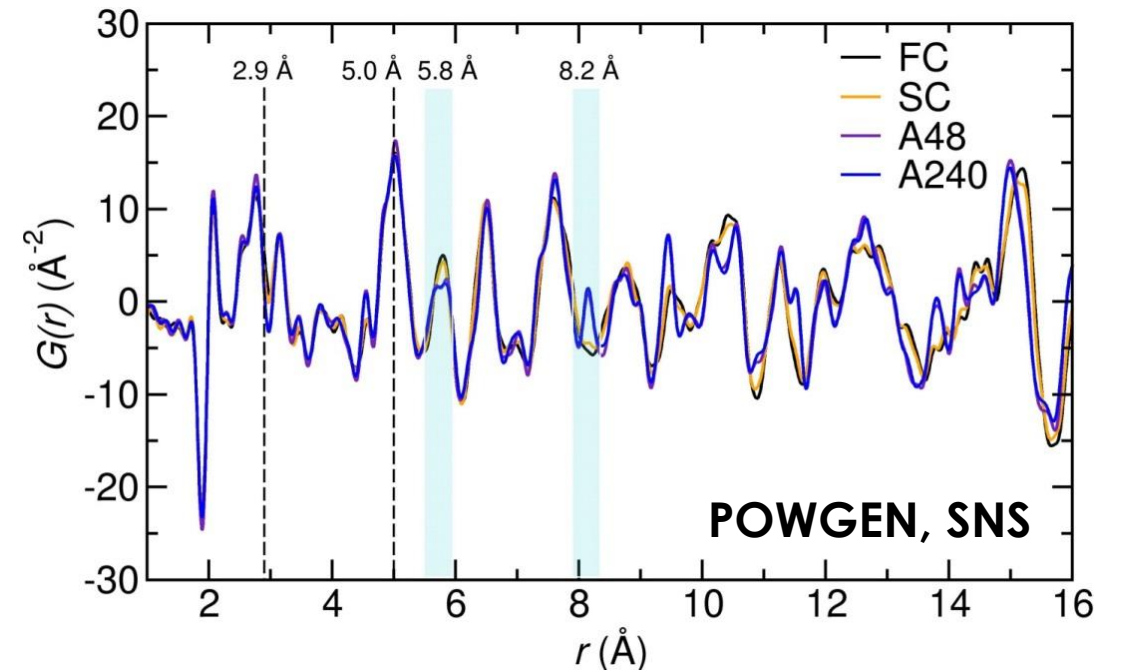
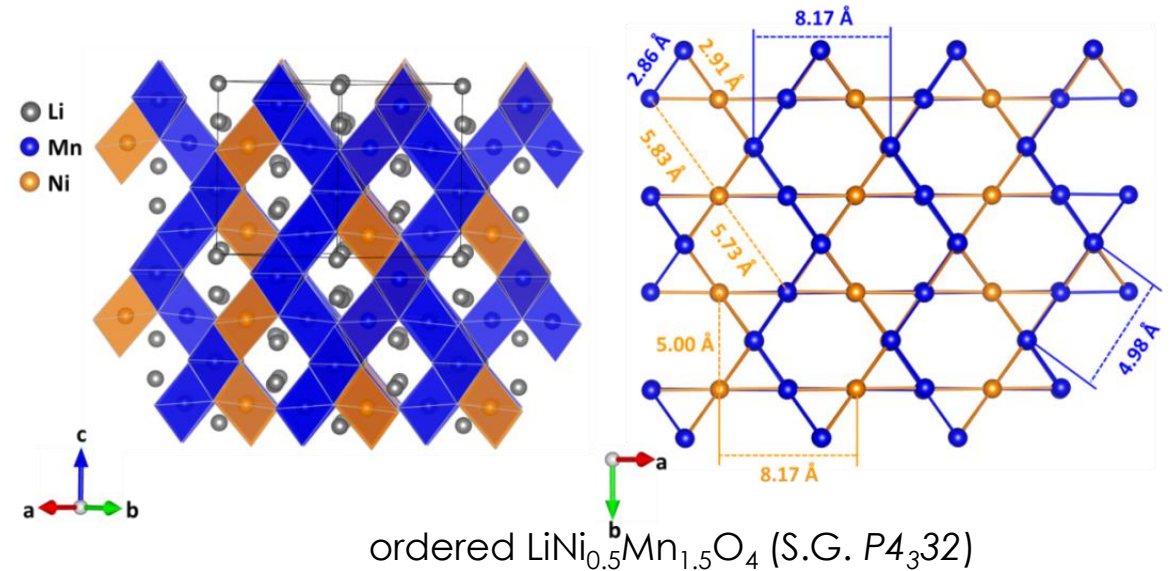
Z. Moorhead-Rosenberg, A. Huq, J. B. Goodenough, & A. Manthiram,  
*Chem. Mater.* (2015) **27**, 6934-6945.

# Example: High Voltage Spinel Cathode ${}^7\text{LiNi}_{0.5}\text{Mn}_{1.5}\text{O}_4$

## A lot can be observed by looking at the PDFs:

- Local atomic structures almost identical up to 5 Å (two nearest B-site neighbors)
- Sample structures diverge after that
- Annealed samples: two distinguishable sets of Ni/Mn pairs at third nearest Ni/Mn neighbor distance
- By fourth nearest Ni/Mn neighbor the samples are distinct

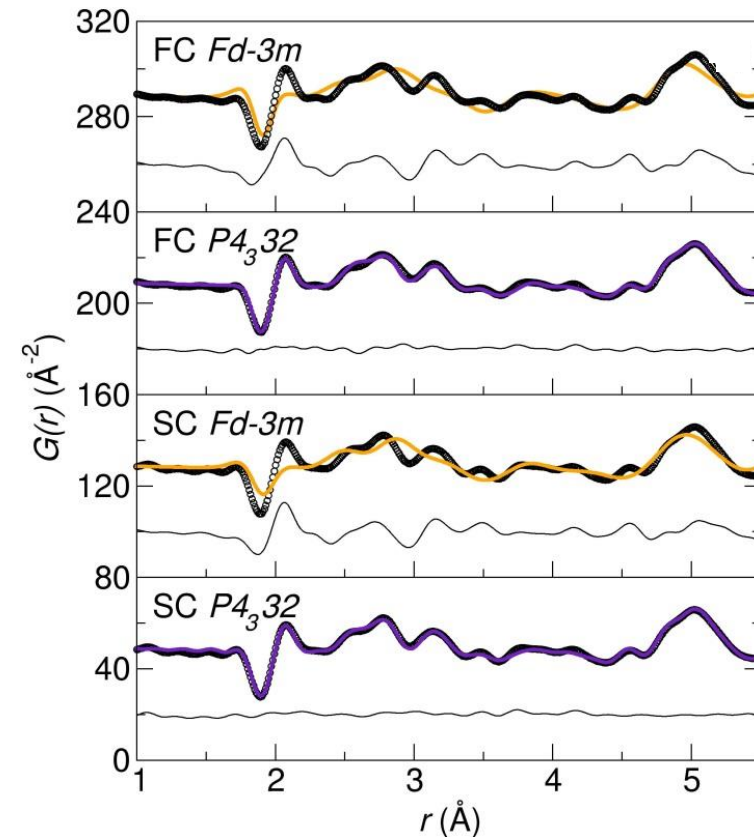
Liu J., Huq A., Moorhead-Rosenberg Z., Manthiram A., Page K., **Nanoscale Ni/Mn Ordering in the High Voltage Spinel Cathode  $\text{LiNi}_{0.5}\text{Mn}_{1.5}\text{O}_4$** , *Chemistry of Materials*, 28, (2016) 6817–6821.



# Example: High Voltage Spinel Cathode ${}^7\text{LiNi}_{0.5}\text{Mn}_{1.5}\text{O}_4$

Additional information from modeling the local structure

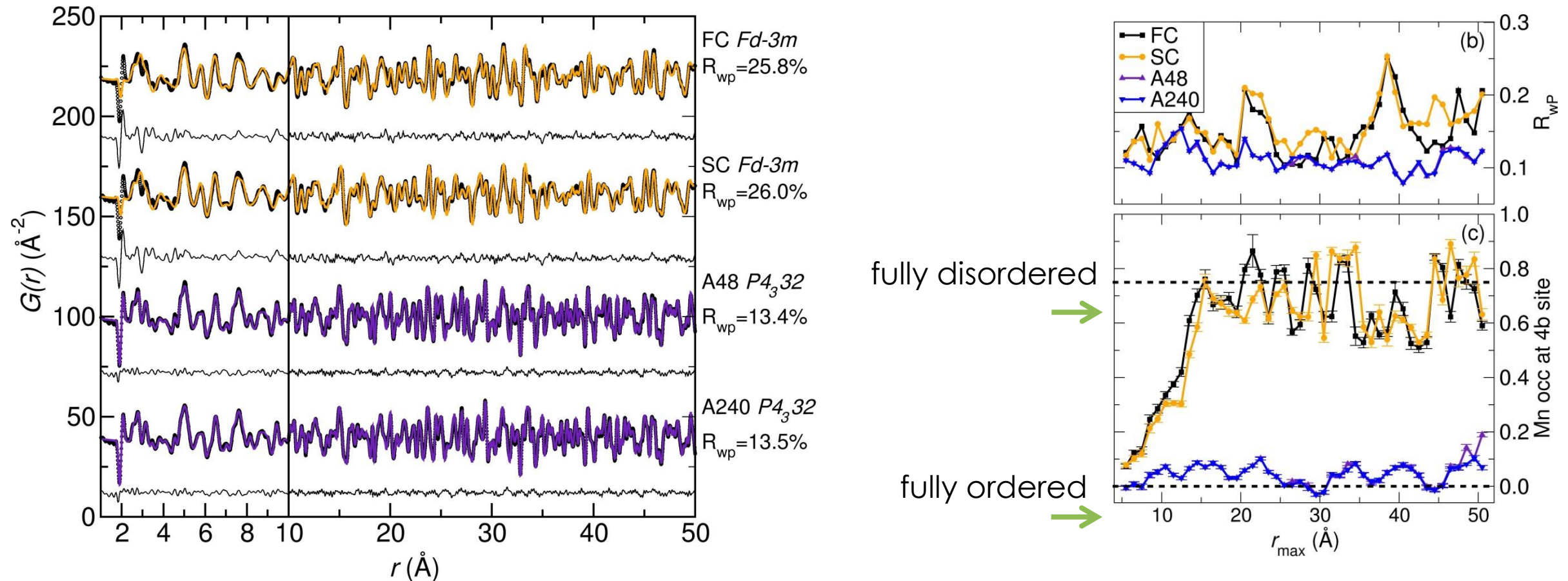
- Least squares refinements of the ordered structure model ( $P4_332$ ) were carried out in a manner that  $4b$  site (Ni site) and  $12d$  site (Mn site) occupancies are allowed to refine simultaneously but with site multiplicity constraints
- Over 1 to 5 Å range the ordered models ( $P4_332$ ) with Mn/Ni site mixing provide much better fits for local PDF profiles
- **Ni/Mn are locally well-ordered in the long-range “disordered” samples**
- **But up to what length scale?**



J. Liu, A. Huq, Z. Moorhead-Rosenberg, A. Manthiram, and K. Page, **Nanoscale Ni/Mn ordering in the high voltage spinel cathode  $\text{LiNi}_{0.5}\text{Mn}_{1.5}\text{O}_4$** , *Chemistry of Materials*, 28, 19, 6817–6821, 2016.

# Example: High Voltage Spinel Cathode ${}^7\text{LiNi}_{0.5}\text{Mn}_{1.5}\text{O}_4$

15 Å correlation length scale for SRO

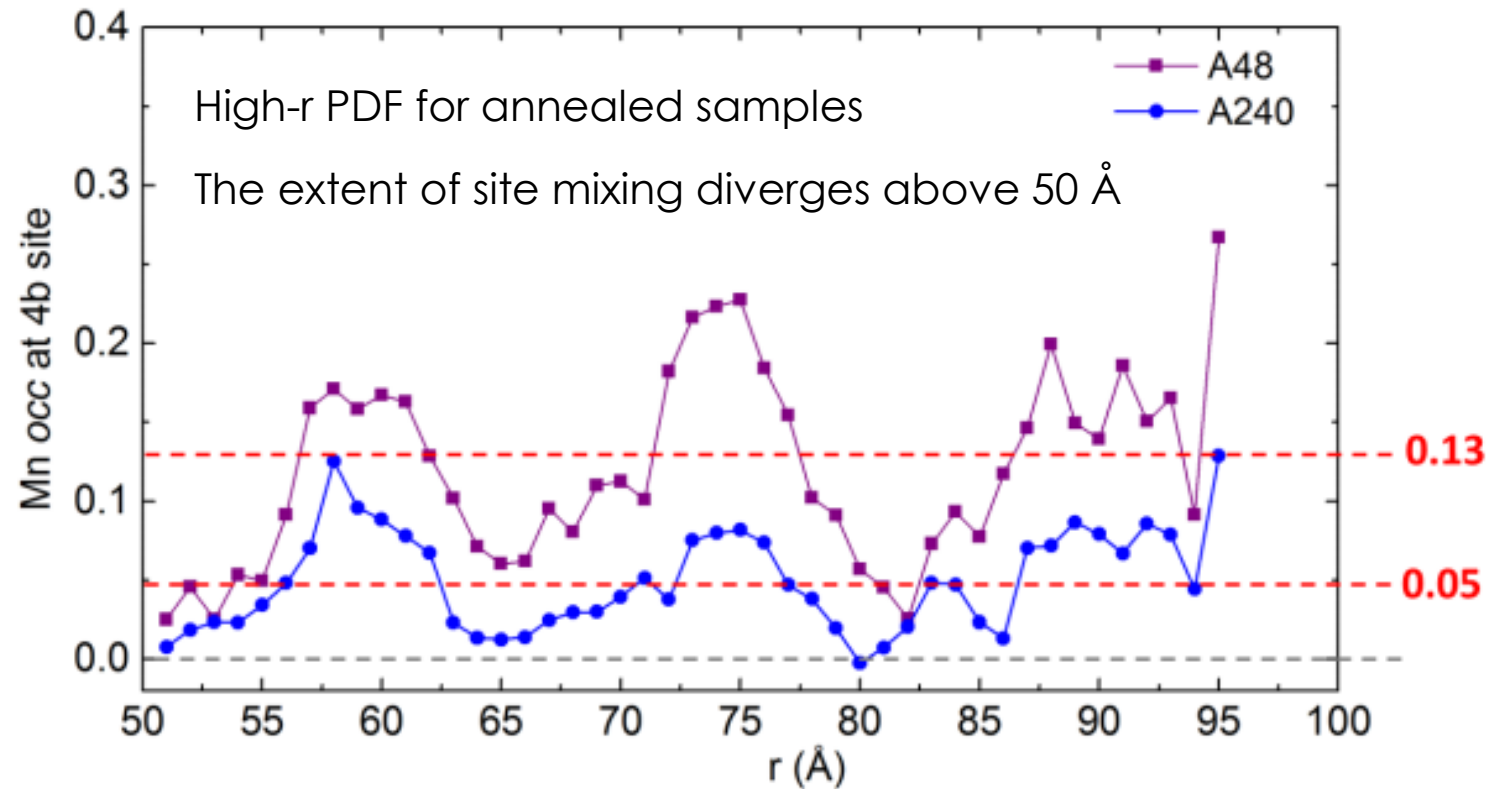


- 5% site mixing in the A48 and A240 patterns throughout the entire range
- FC and SC samples are nearly fully disordered at pair distances beyond 15.5 Å



# Example: High Voltage Spinel Cathode ${}^7\text{LiNi}_{0.5}\text{Mn}_{1.5}\text{O}_4$

Fit the PDFs within a 4.5 Å “box” in 1 Å steps (a “box-car” refinement)



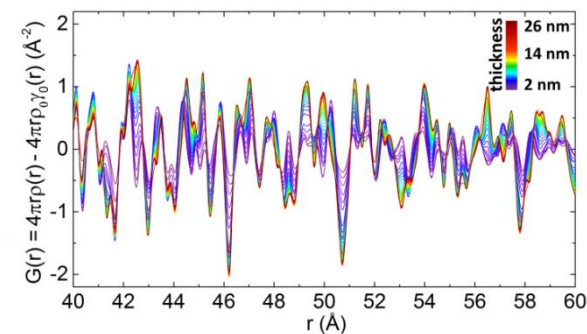
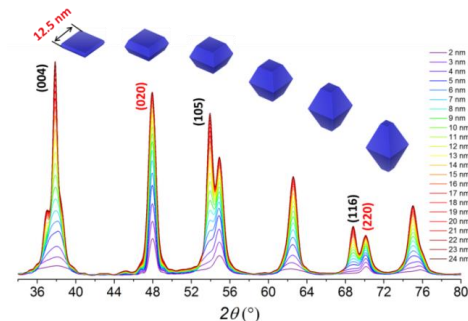
***Spinel cathode materials are distinguished by their unique correlation length scales for chemical short range ordering***

# Nanomaterial structure *via* PDF

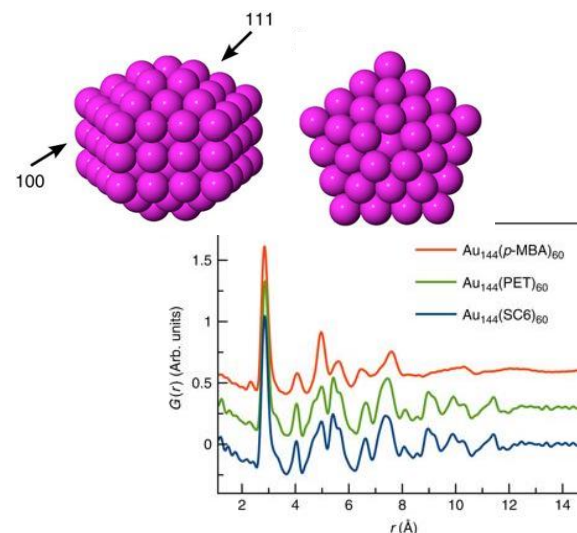
- Finite size/shape effects
- Surface/Interface structure
- Nanostructure polymorphs
- Growth and transformation



K. W. Chapman, P. J. Chupas, and T. M. Nenoff, **Radioactive Iodine Capture in Silver-Containing Mordenites through Nanoscale Silver Iodide Formation**, *J. Am. Chem. Soc.* 132, 8897 (2010).



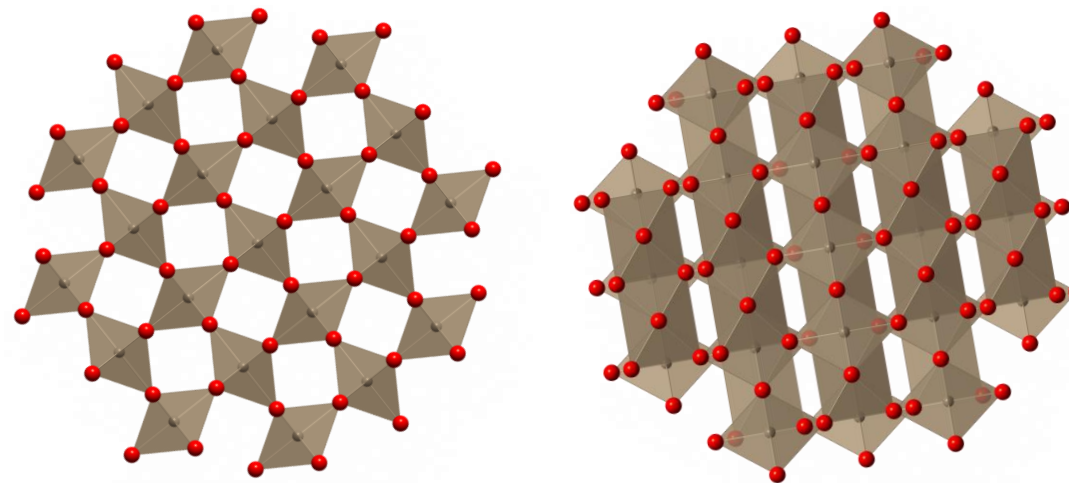
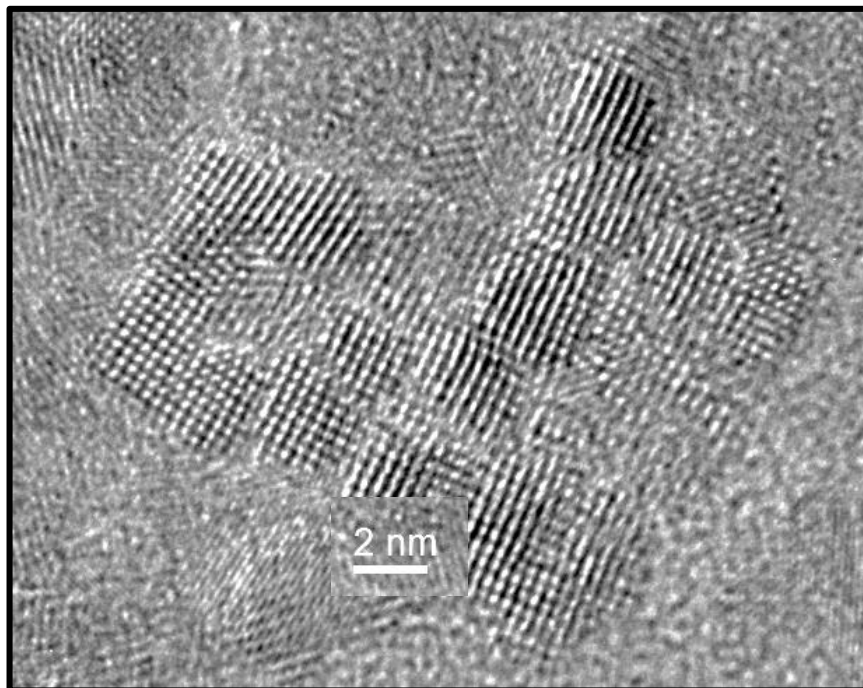
J. Liu, D. Olds, R. Peng, L. Yu, G. S. Foo, S. Qian, J. Keum, B. S. Guiton, Z. Wu, and K. Page, **Quantitative analysis of the morphology of {101} and {001} faceted anatase TiO<sub>2</sub> nanocrystals**, *Chem. Mater.* 29, 5591–5604 (2017).



K. M. O. Jensen, P. Juhas, M. A. Tofanelli, C. L. Heinecke, G. Vaughan, and C. J. Ackerson, **Polymorphism in magic-sized Au<sub>144</sub>(SR)<sub>60</sub> clusters**, *Nature Communications* 7, 11859 (2016).

# Example: SnO<sub>2</sub> Nanocrystals

~2 nm SnO<sub>2</sub> (cassiterite) nanocrystals capped with H<sub>2</sub>O/OH or D<sub>2</sub>O/OD groups



TGA suggests 2 steps dehydration.

How many layers of water are at the surface?

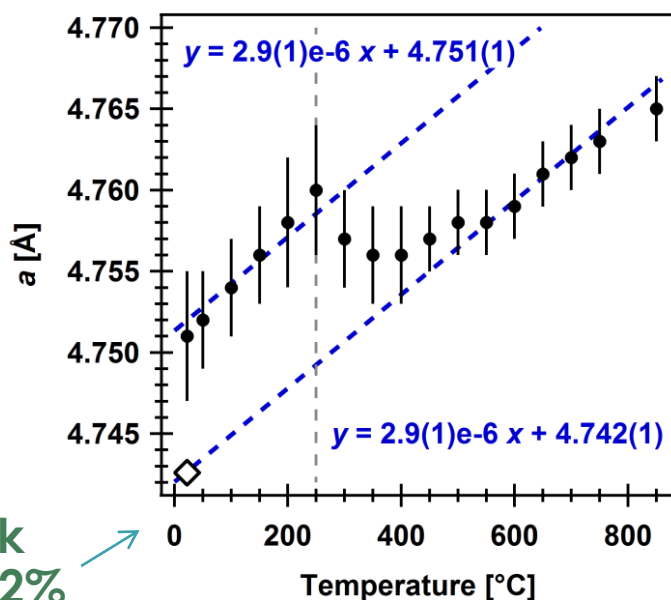
How is water bonded to surfaces?

What are the dynamics of dehydration?

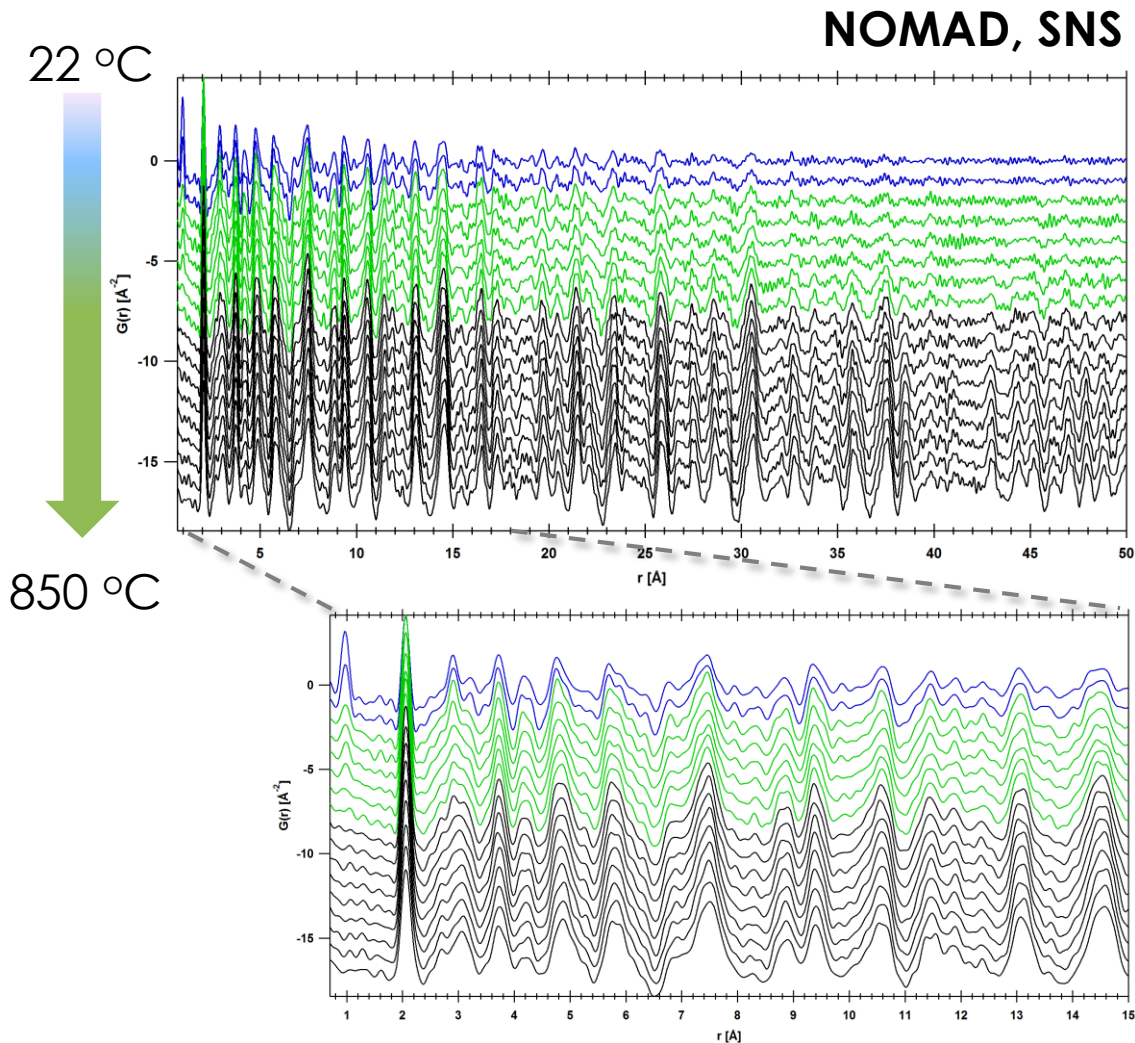
H.-W. Wang, D. J. Wesolowski, T. Proffen, L. Vleck, W. Wang, L. F. Allard, A. I. Kolesnikov, M. Feygenson, L. M. Anovitz, and R. L. Paul, **Structure and stability of SnO<sub>2</sub> nanocrystals and surface-bound water species**, *J. Am. Chem. Soc.*, 135, 6885-6895, 2013.

# Example: SnO<sub>2</sub> Nanocrystals

- 22 to 50 °C: L<sub>1</sub> + L<sub>2</sub> + L<sub>3</sub>,
- 50 to 350 °C: L<sub>1</sub> + L<sub>2</sub>
- 400 to 850 °C: SnO<sub>2</sub> grain growth



Bulk  
~0.2%  
smaller

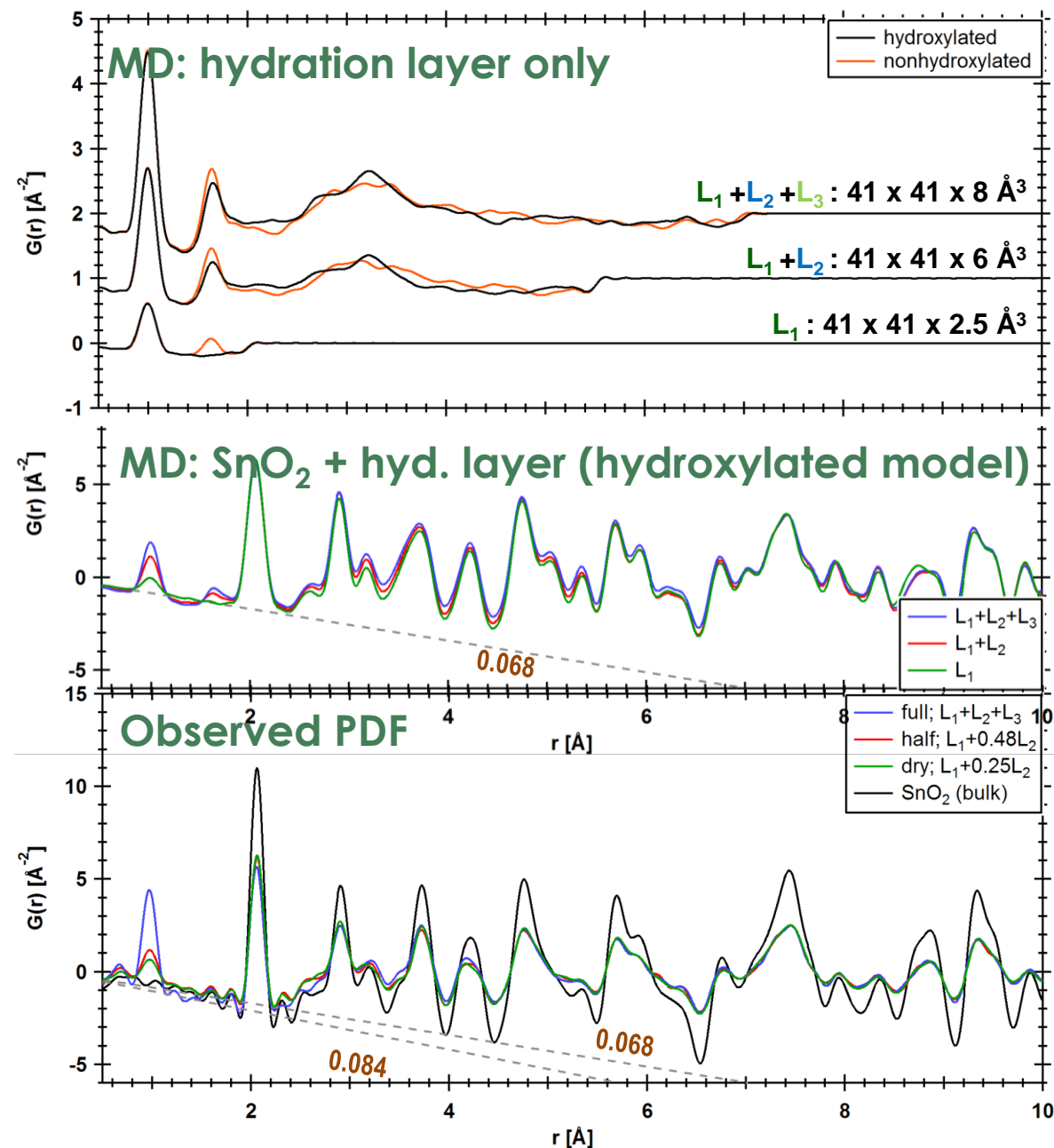
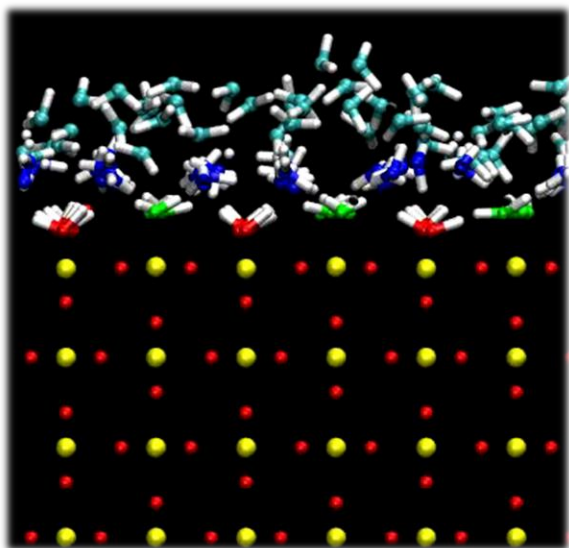


*In situ* dehydration answers these questions and indicates that water plays a key role in stabilizing the nanocrystals.

# MD and PDF

Data is compared to Molecular Dynamics Simulation PDFs for nonhydroxylated and hydroxylated models:

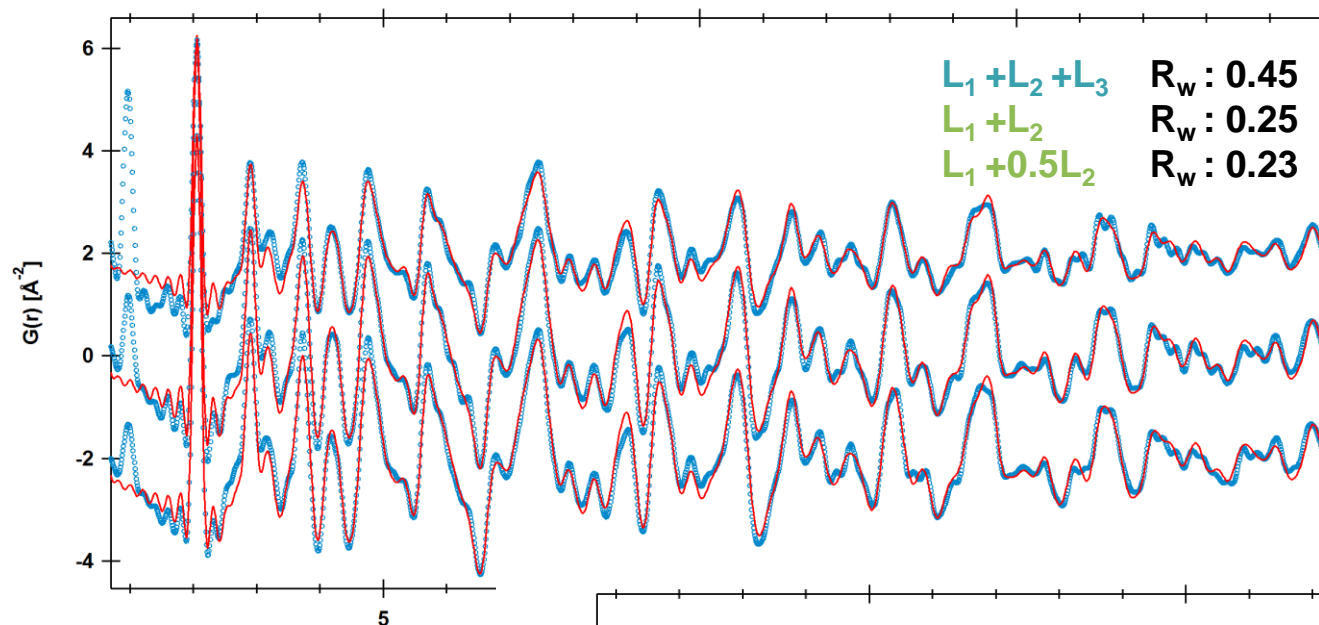
Box size:  $41 \times 41 \times 23 \text{ \AA}^3$ ; 2592 atoms; # density =  $0.068 \text{ \AA}^{-3}$ ;  $U_{\text{iso}} = 0.003 \text{ \AA}^2$



# Example: SnO<sub>2</sub> Nanocrystals

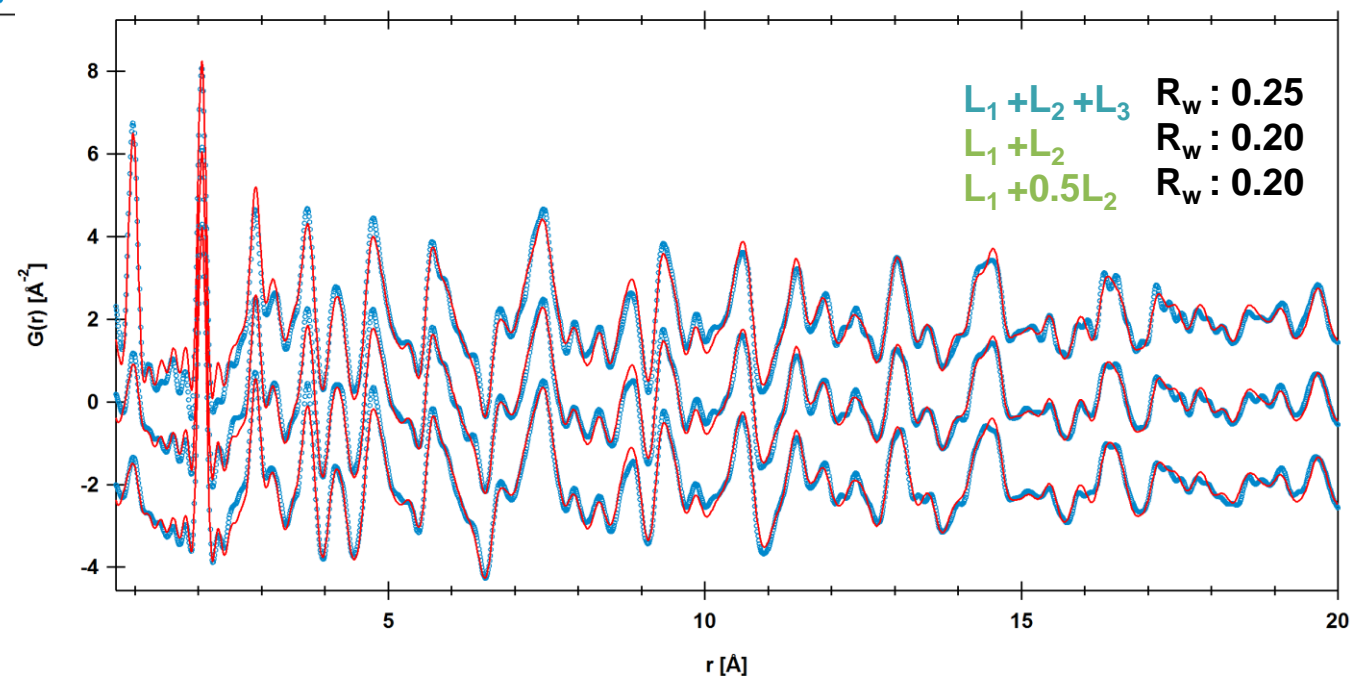
## Single phase model:

SnO<sub>2</sub> bulk structure,  
refined particle  
size = ~47 Å

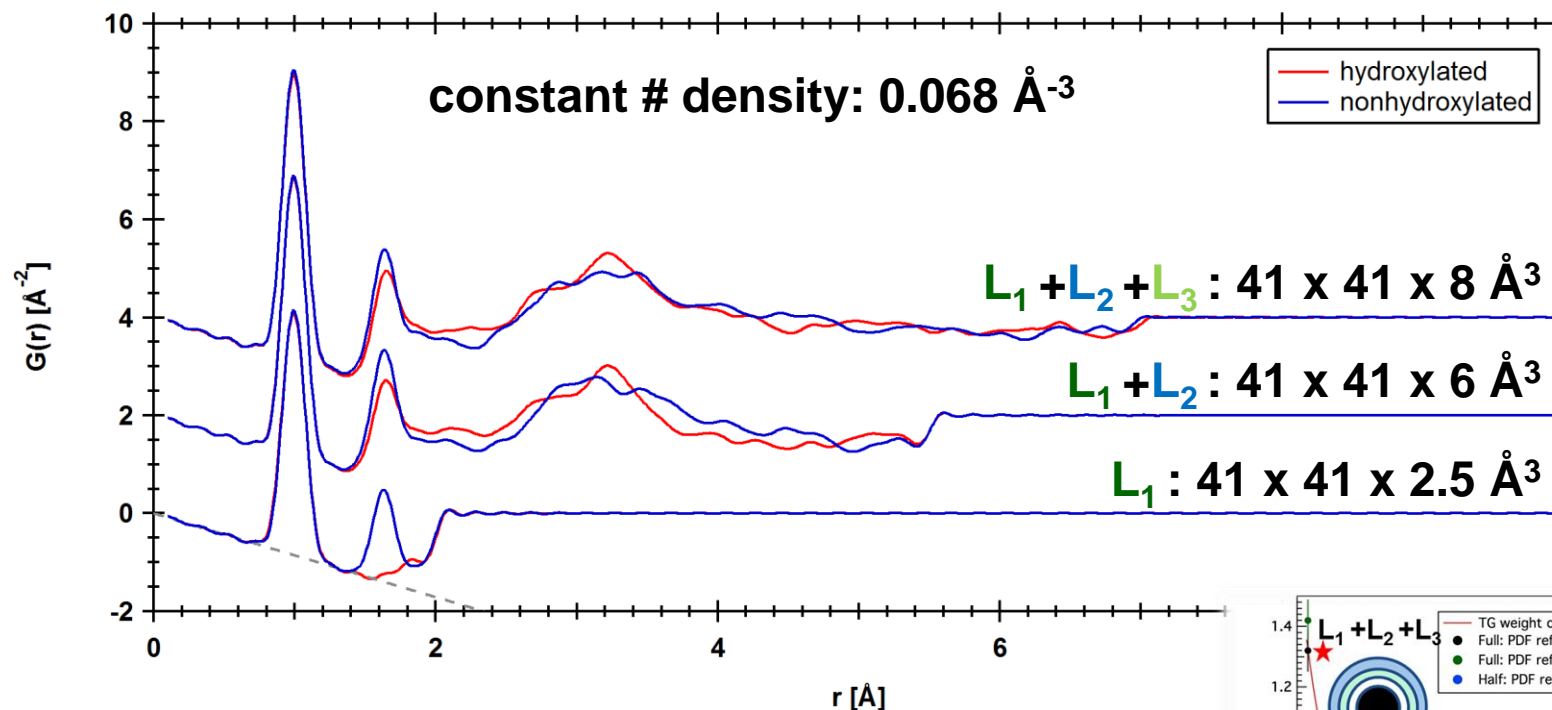


## Two phase model:

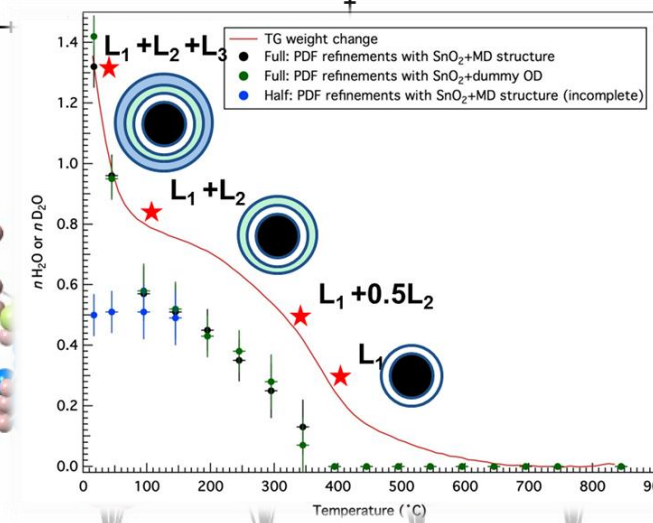
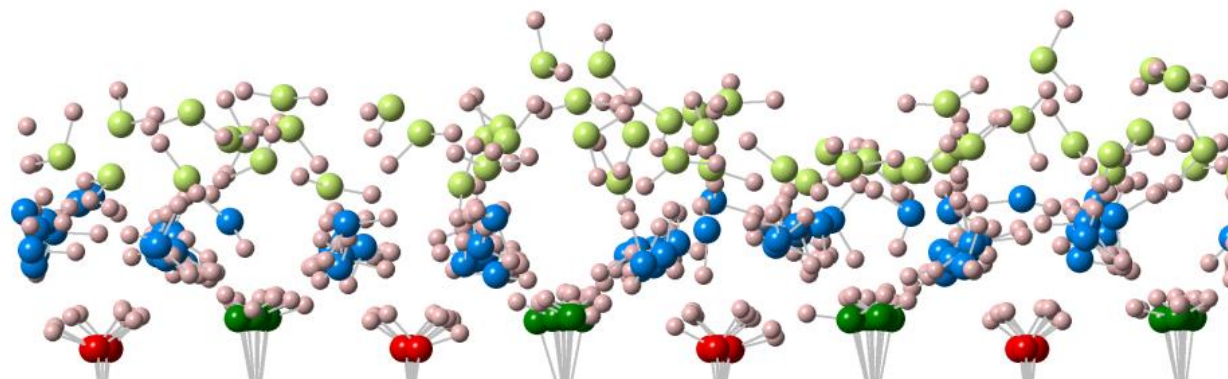
SnO<sub>2</sub> bulk +  
layered MD  
water structure



# Example: SnO<sub>2</sub> Nanocrystals

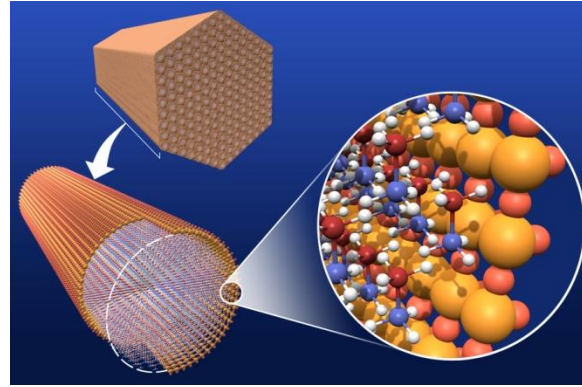


H.-W. Wang, D. J. Wesolowski, T. Proffen, L. Vlcek, W. Wang, L. F. Allard, A. I. Kolesnikov, M. Feygenson, L. M. Anovitz, and R. L. Paul, **Structure and stability of SnO<sub>2</sub> nanocrystals and surface-bound water species**, *J. Am. Chem. Soc.*, 135, 6885-6895, 2013.



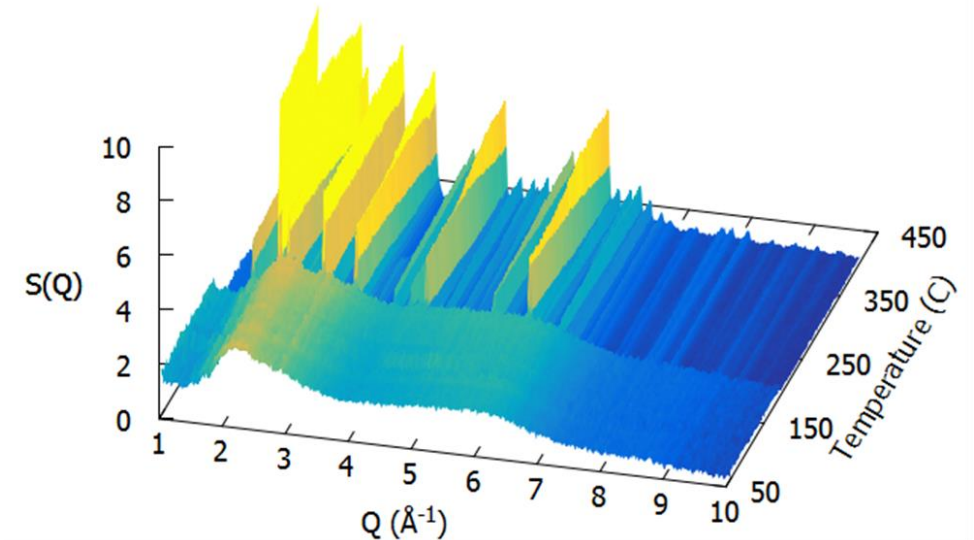
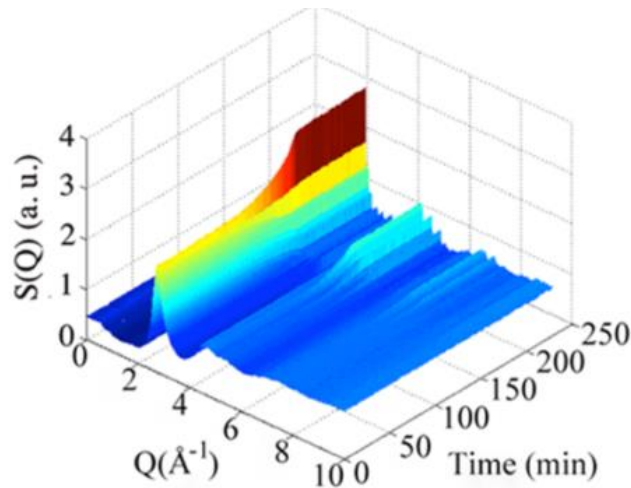
# Amorphous structures via PDF

- Glasses
- Liquids
- Concretes
- Adsorbed/absorbed gases
- *etc.*



H. Kim, T. Proffen, P. J. Chupas, A. Karkamkar, N. J. Hess, and T. Autrey, **Determination of structure and phase transition of light element nanocomposites in mesoporous silica: case study of  $\text{NH}_3\text{BH}_3$  in MCM-41**, *J. Am. Chem. Soc.* 131, 13749-13755 (2009).

S. Lan, X.. Wei, J. Zhou, Z. Lu, X. Wu, M. Feygenson, J. Neuefeind, X. Wang, **In situ study of crystallization kinetics in ternary bulk metallic glass alloys with different glass forming abilities**, *Applied Physics Letters*, 105, 201906 (2014).

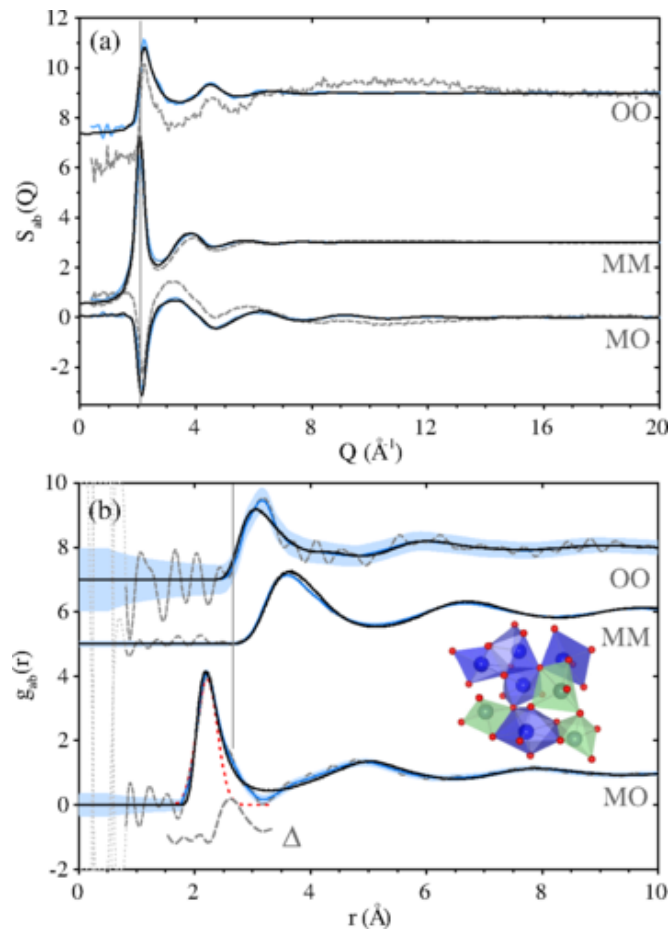


H.-W. Wang; L. L. Daemen, M. C. Cheshire, M. K. Kidder, A. G. Stack, L. F. Allard, J. Neuefeind, D. Olds, J. Liu, and K. Page, **Synthesis and structure of synthetically pure and deuterated amorphous (basic) calcium carbonates**, *Chem. Commun.*, 53, 2942-2945 (2017).



# Example: Molten Rare Earth Oxides

Understanding of oxide liquids is essential in nuclear meltdown scenarios, evolution of planetary bodies, glass formation and crystal nucleation.

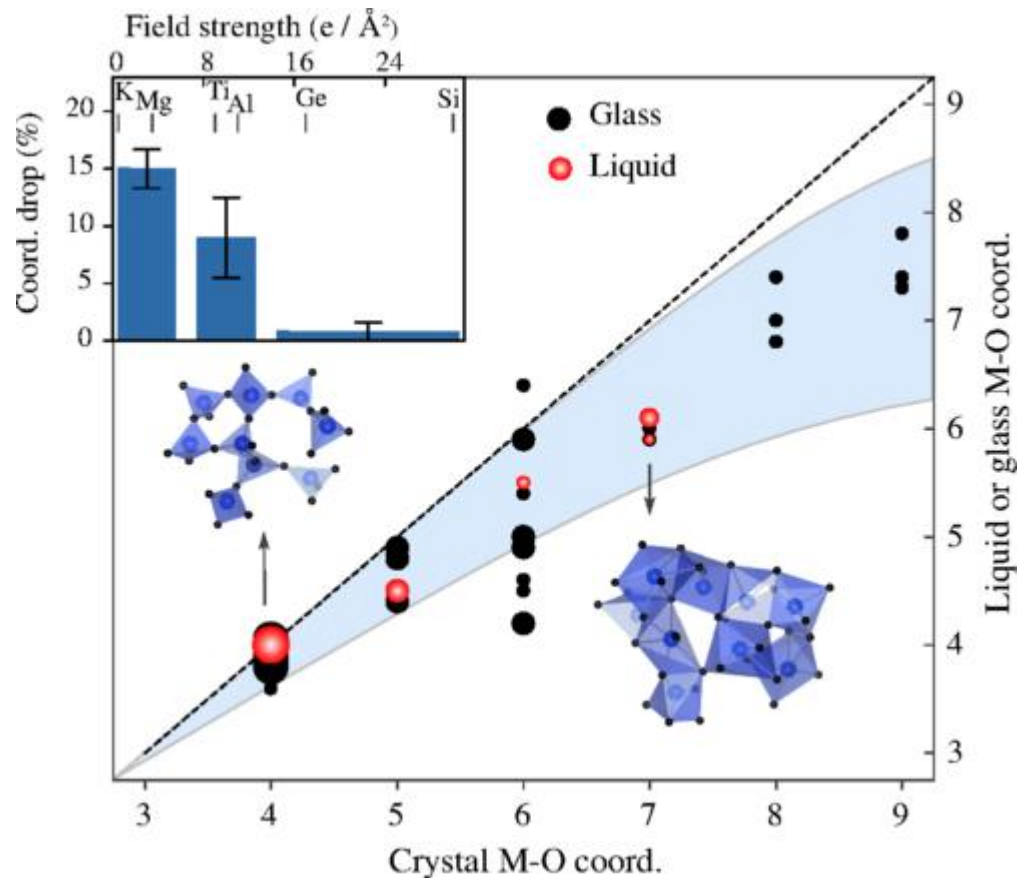


NOMAD, SNS / 11-ID-C, APS

- Partial pair distribution functions for molten  $M_2O_3$  ( $M = Y$  or  $Ho$ ) at 2870 K
- Aerodynamic levitation and laser heating in conjunction with neutron and high energy X-ray diffraction
- Complete set of partial pair distribution functions obtained for the first time for a high temperature oxide melt

Skinner L.B., Benmore C.J., Weber J.K.R., Du J., Neuefeind J., Tumber S.K., and Parise, J.B., **Low Cation Coordination in Oxide Melts**, *Physical Review Letters*, 112, 157801, (2014).

# Example: Molten Rare Earth Oxides



- Metal oxygen first shell coordination number is significantly lower than in the corresponding crystal
- Trend persists in other oxide melts
- Reduction in coordination number is correlated with the ionic field strength of the constituting metal ion

# A Few Experimental Considerations

- Measurements and corrections
- Resolution and range effects
- Instruments



# Total Scattering Structure Function

Structure function, determined from the scattering intensity/differential cross section:

coherent scattering intensity (corrected)      scattering length (neutrons) or atomic form factor (x-rays)

$$s(Q) = \frac{I_{coh}(Q) - \sum c_i |b_i|^2}{|\sum c_i b_i|^2} + 1 \quad Q = \frac{4\pi \sin \theta}{\lambda}$$

*Corrected for:* Container & background scattering, self-absorption, etc.

*Normalized by:* Incident flux, number of atoms, square of the scattering length/form factor

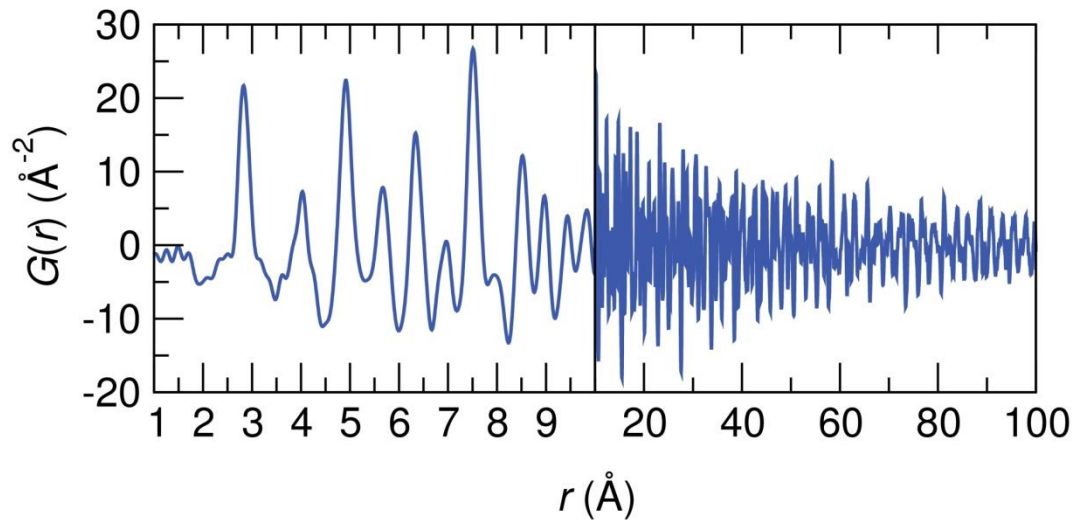
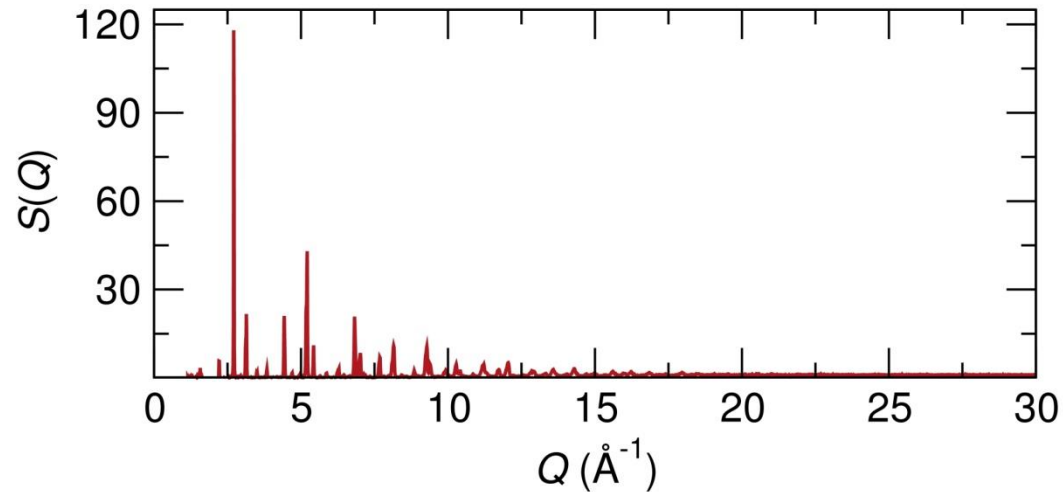
For unambiguous derivation of this derivation and relationship to other forms:

C. Farrow and S. J. L. Billinge, *Acta Cryst.* (2009) A65, 232–239.

D. A. Keen, *J. Appl. Cryst.* 34 (2001) 172-177.

# The Experimental PDF

The Sine Fourier transform of the total (Bragg and diffuse) scattering



The total scattering structure factor:  $S(Q)$

$$S(Q) = \frac{I_{coh}(Q) - \sum c_i |b_i|^2}{|\sum c_i b_i|^2} + 1$$



Sine Fourier transform

The Pair Distribution Function (PDF):  $G(r)$

$$G(r) = \frac{2}{\pi} \int_{Q_{min}}^{Q_{max}} Q[S(Q) - 1] \sin(Qr) dQ$$

# Obtaining High Quality PDFs

- (1) High maximum momentum transfer ( $Q_{\max}$ )
- (2) Good Q-resolution,  $dQ$
- (3) Good counting statistics
- (4) Low (and stable) instrument background

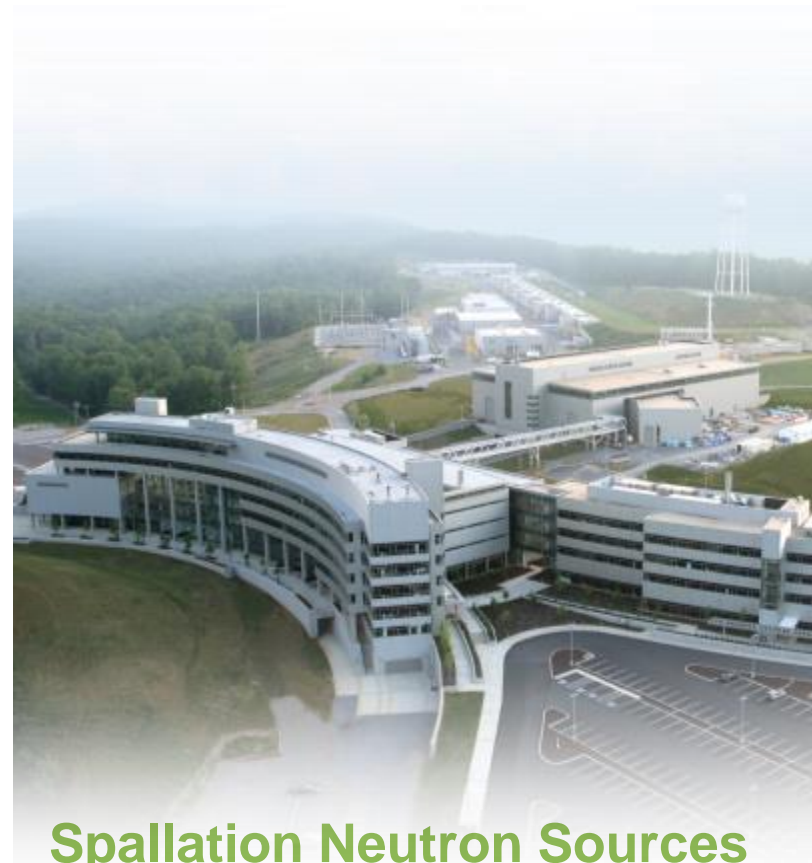
An ideal measurement would have no contribution from the instrument resolution

For PDF: a wide Q range and high flux is balanced with resolution

**Synchrotron sources**  
(high energy X-rays)

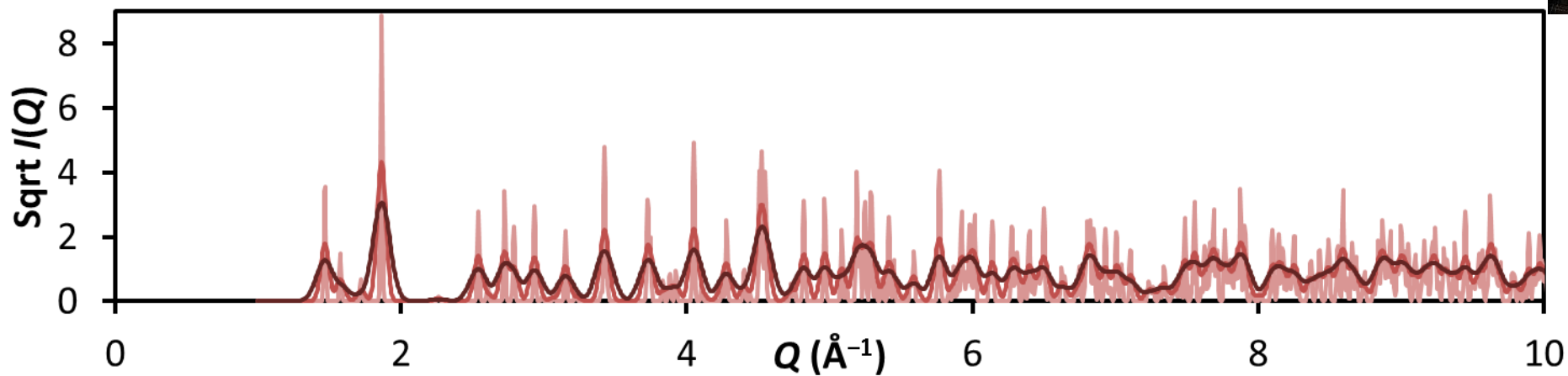
or

**Spallation Neutron Sources**  
(reactor neutron energies are too low)

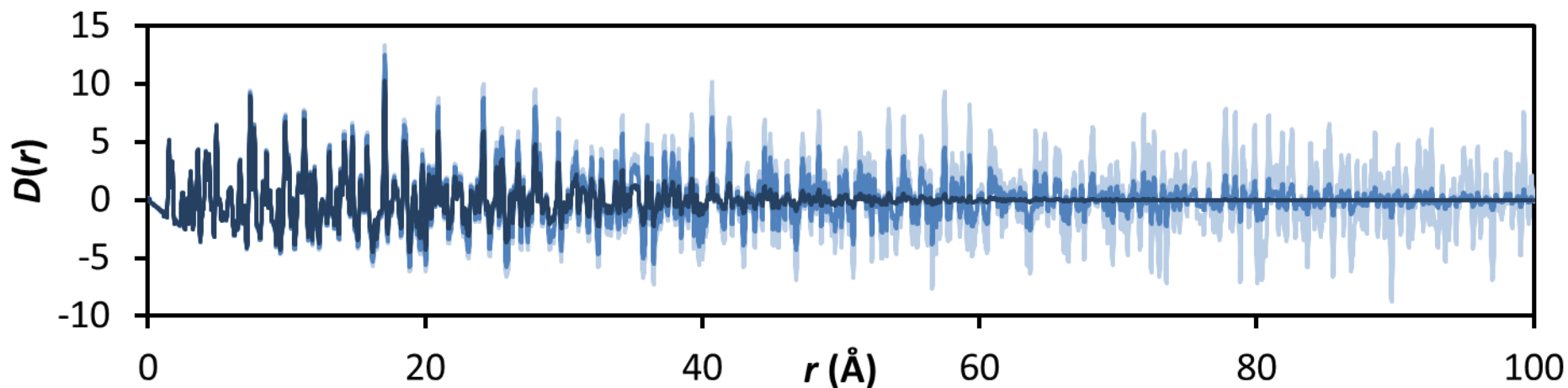


# Resolution Effect

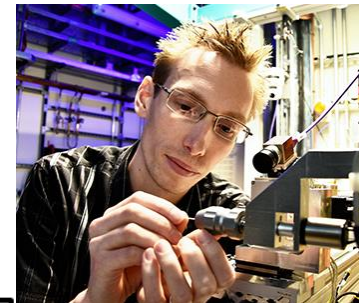
Reciprocal space: Peak width,  $dQ$



Real space: Damping with  $r$



Courtesy of  
Phil Chater,  
Diamond Light  
Source

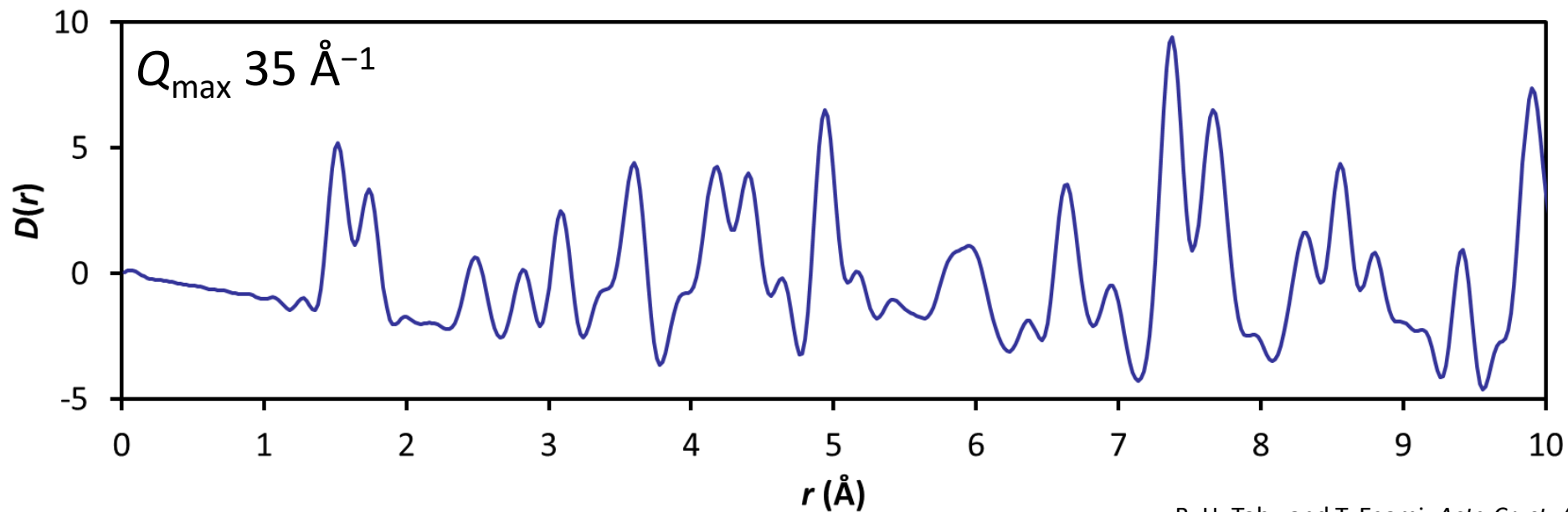
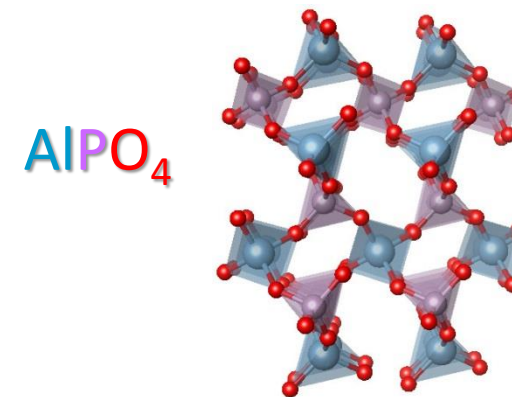
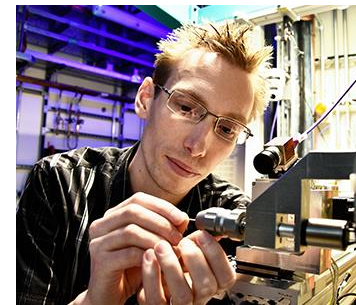


# $Q_{\max}$ Effect

$\Delta r$  resolution of a PDF is dominated by  $Q_{\max}$

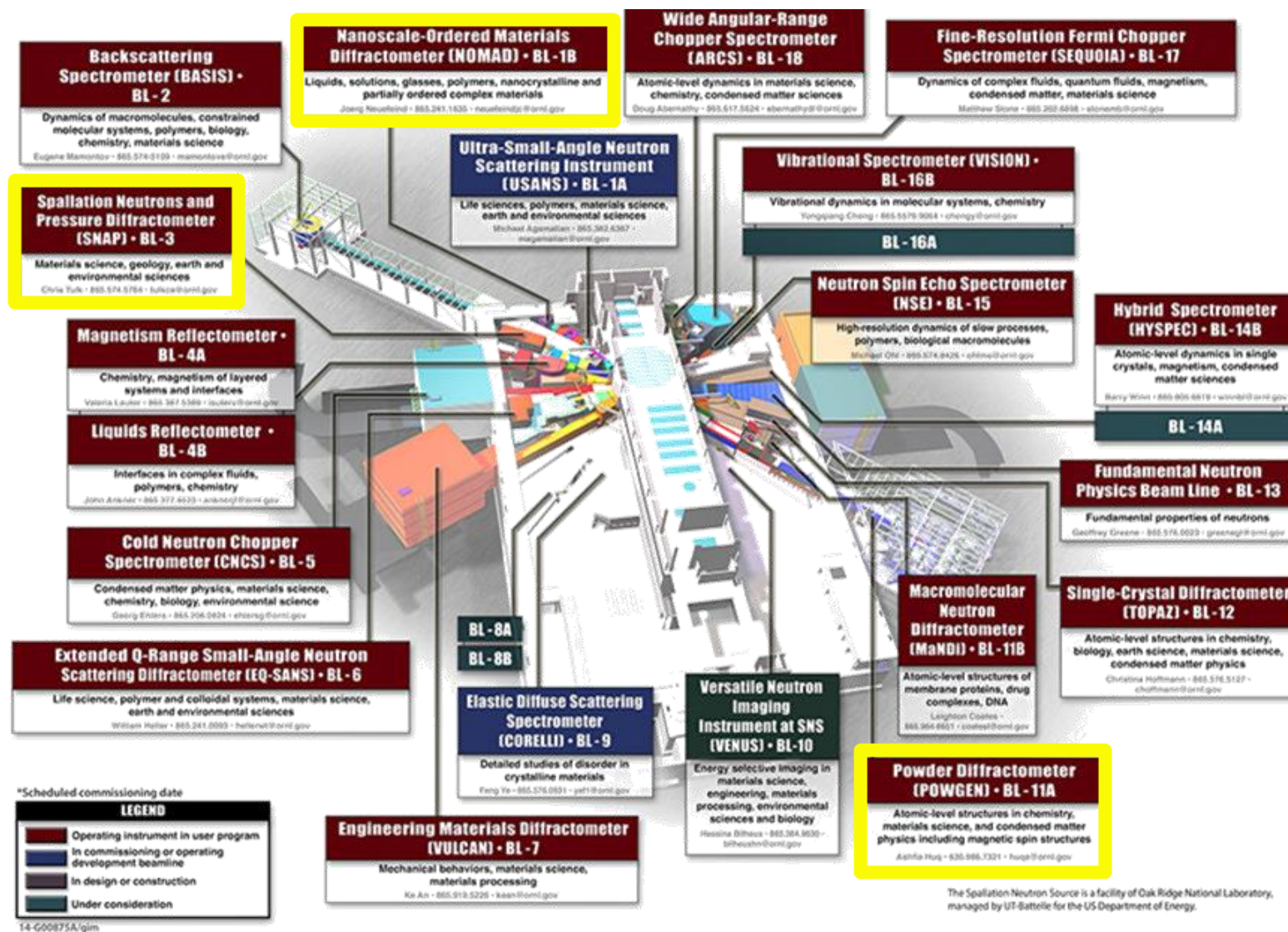
- $Q = 2\pi/d = 4\pi\sin\theta/\lambda$
- $\Delta r \approx 2\pi/Q_{\max}$

Courtesy of  
Phil Chater,  
Diamond Light  
Source





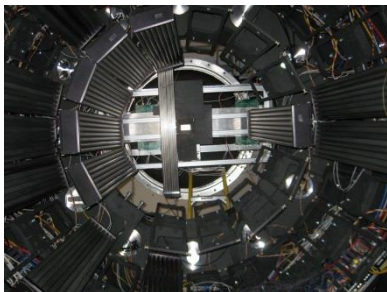
# Instruments at SNS for PDF studies



The Spallation Neutron Source is a facility of Oak Ridge National Laboratory, managed by UT-Battelle for the US Department of Energy.

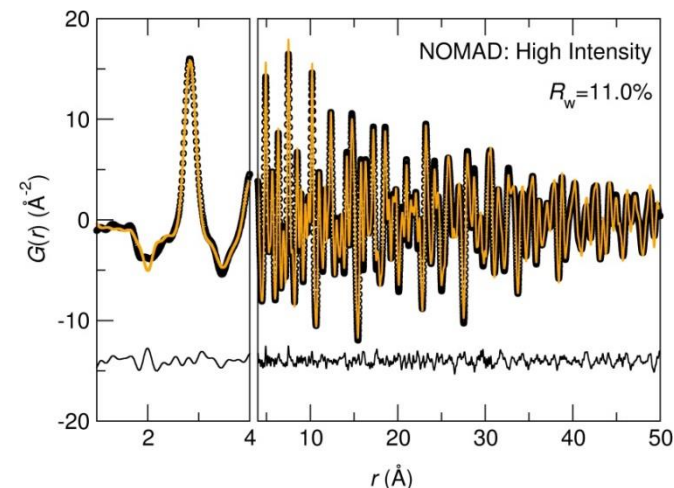
# TOF Diffraction and Total Scattering at SNS

Pair Distribution Function (PDF) methods follow local atomic bonding configurations, intermediate structure, and correlation length scale- *regardless of a material's long-range structure*



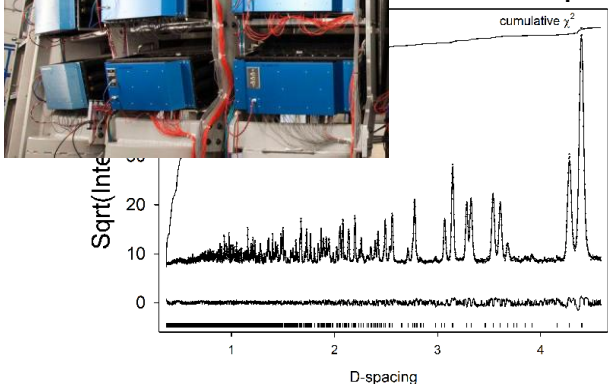
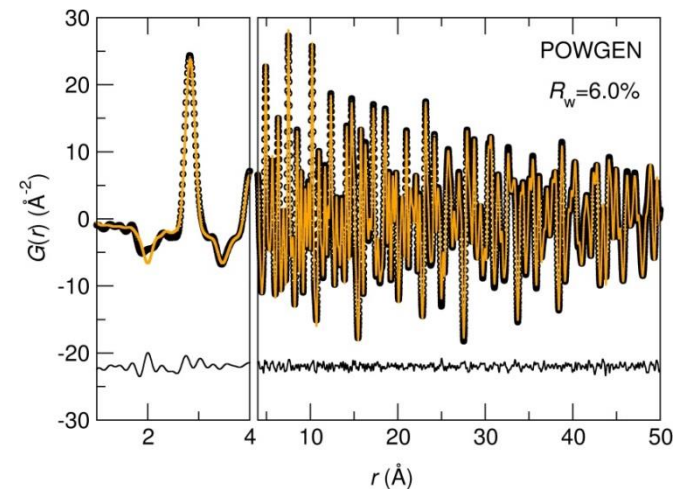
Typical **NOMAD** data can be collected for **30 - 100 mg** of sample in a 3 mm quartz capillary in **~1 hour**

*high intensity* diffraction and PDF for small samples and in situ studies on amorphous, nanostructured, and crystalline materials



Typical **POWGEN** data can be collected for **~3 - 10 g** of sample in a 6 mm vanadium canister in **~3 hours**

*high resolution* diffraction and PDF of crystalline materials

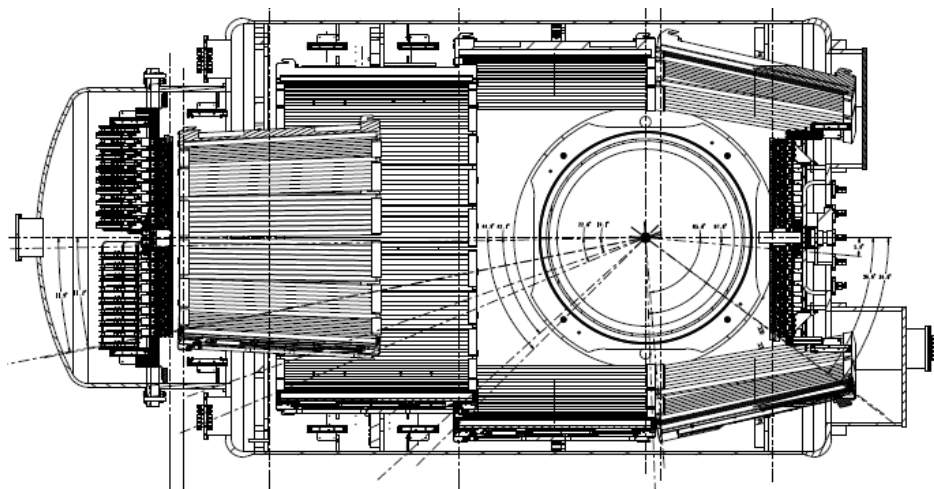


FOUNDATIONS  
ADVANCES  
ISSN 2053-2733

[feature articles](#)

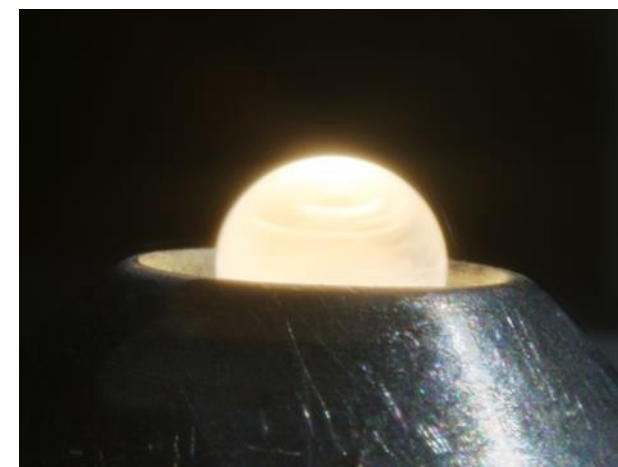
Precise implications for real-space pair distribution function modeling of effects intrinsic to modern time-of-flight neutron diffractometers

# Nanoscale-Ordered Materials Diffractometer (NOMAD)



Neufeind J., Feygenson M., Carruth J., Hoffmann R., Chipley K.,  
**The Nanoscale Ordered MATERIALS Diffractometer NOMAD at the  
Spallation Neutron Source SNS**, *Nuclear Instruments and Methods  
B*, 287, 68-75, (2012).

- Large bandwidth of neutron energies
- Extensive detector coverage
- Count rates exceed comparable instruments by one to two orders of magnitude
- Routine Q-range of 0.2 to 40  $\text{\AA}^{-1}$



## Sample Environments

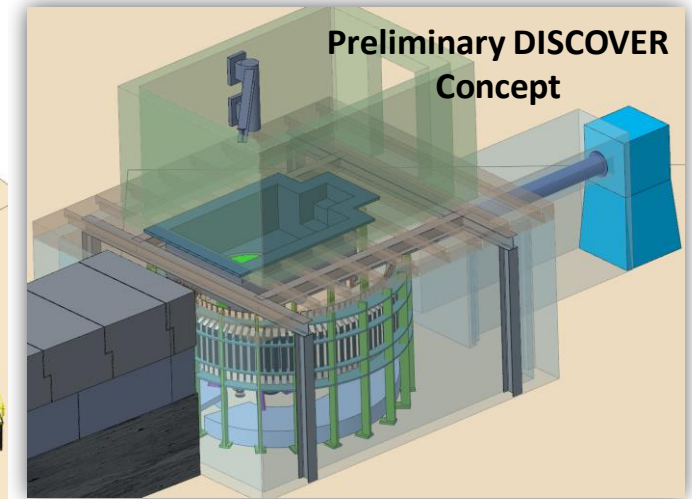
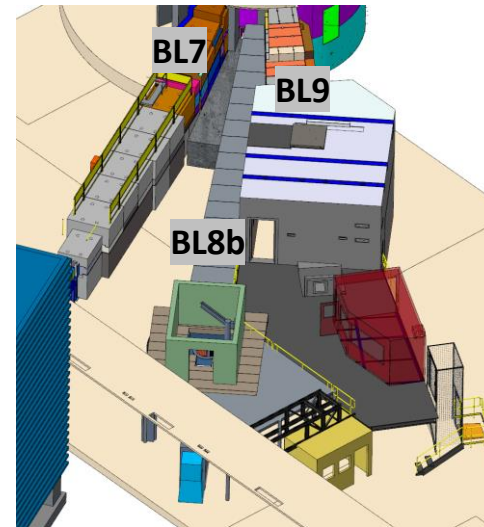
- Sample translation stage (80K- 500K)
- Orange cryostat (2K- 300K)
- ILL furnace (300K- 1400K)
- Aerodynamic levitator (800K- 3500K)
- High precision gas flow cell (RT-800K)
- High voltage set-up (10kV)

# DISCOVER: An SNS Diffractometer for Materials Discovery

## Proposed Instrument for Diffraction and PDF, BL-8b, SNS

Atomic and intermediate range order (and heterogeneity) need to be understood simultaneously and as they evolve to enable materials discovery and design

- What is the connection between global symmetry (i.e. that found over long length-scales) and local symmetry (i.e. that found at the atomic scale)?
- How does order evolve from the atomic to macroscale?
- How can we control it?

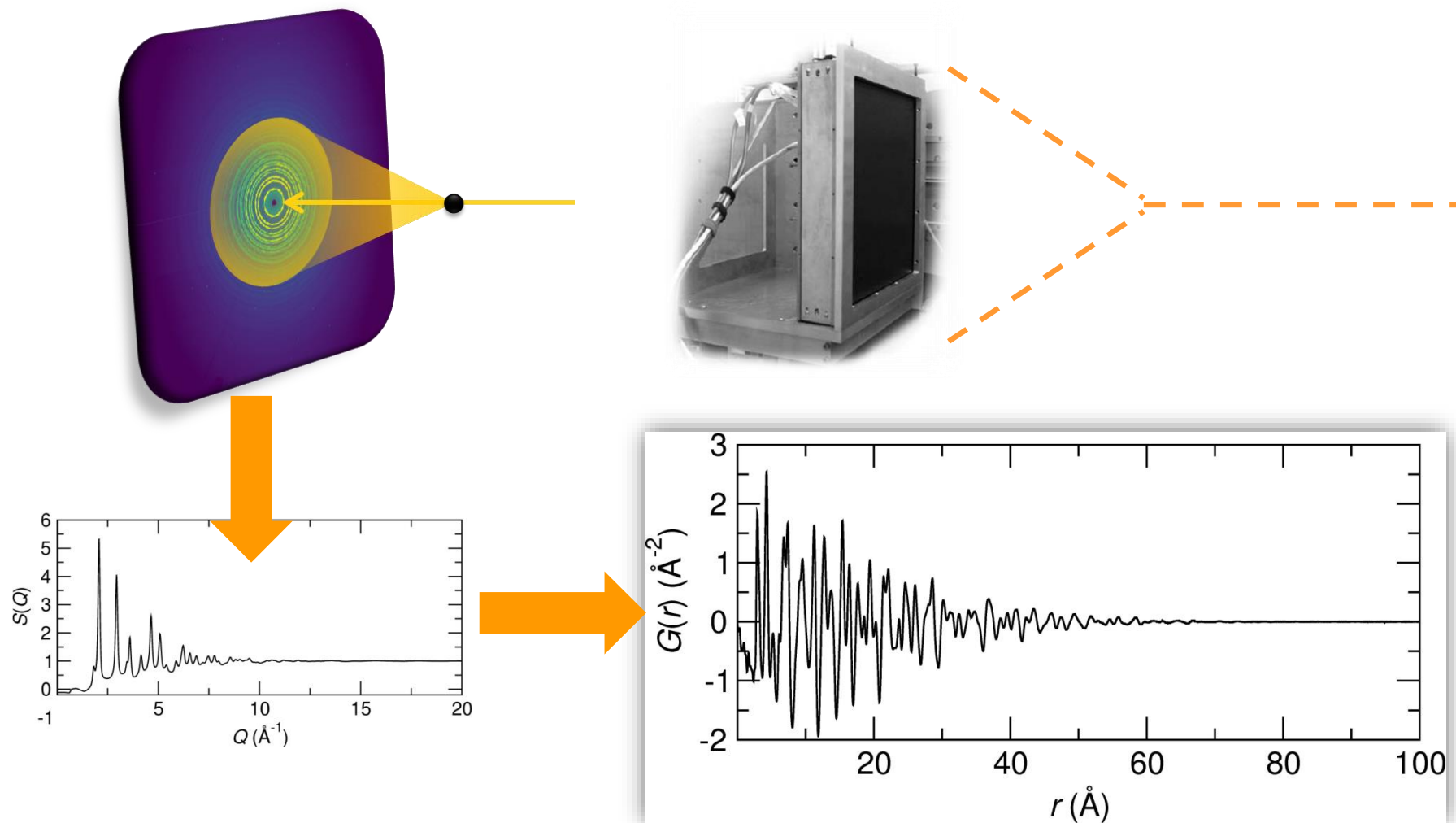


## Day 1 New Capabilities

Simultaneous average (diffraction) and local structure (PDF) determination to follow the evolution of order from atomic to macroscales *in real time* (minutes)

Ability to study hydrogenous materials (particularly ubiquitous in synthesis and catalysis science) with neutrons by separating static from dynamic contributions

# Synchrotron Total Scattering: 2D Amorphous Si Detector



In the US:  
11-ID-B, APS  
PDF, NSLS-II

P. J. Chupas, K. W. Chapman, P. L. Lee, **Applications of an amorphous silicon-based area detector for high resolution, high sensitivity and fast time-resolved pair distribution function measurements**, *J. Appl. Crystallogr.* 40, 463, 2007. <http://dx.doi.org/10.1107/S0021889807007856>

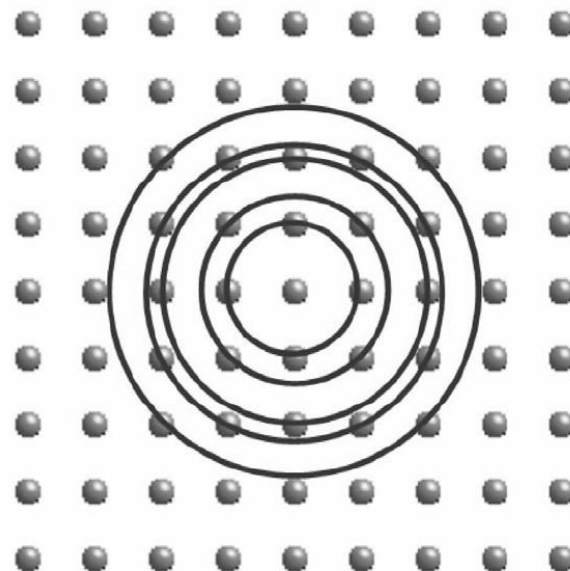
# Modeling a PDF

- Calculating a PDF from a model
- Available software
- Nanoparticle shape effects

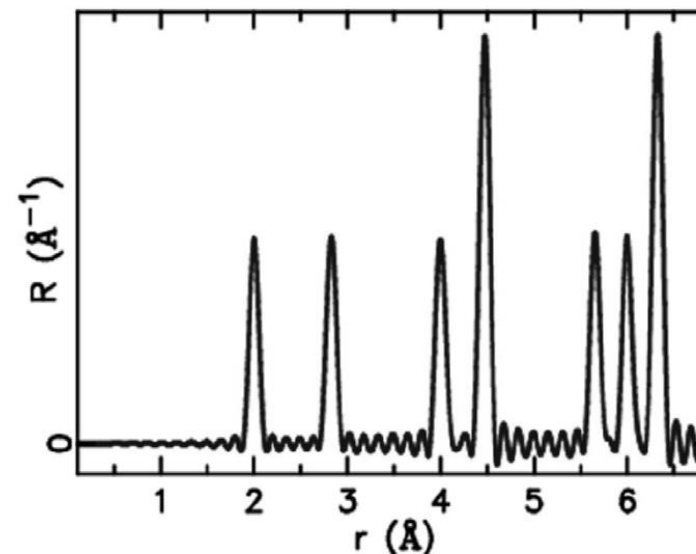


# Pair Distribution Function

Based on the *radial distribution function* (RDF):



S.J.L Billinge, *Z. Kristallogr. Suppl.*,26,17 (2007)



Atomic PDF (PDFFIT notation):

$$G(r) = 4\pi r [\rho(r) - \rho_0]$$

atomic form factors  
( $Z$  for x-rays,  $b$  for neutrons)

$$G(r) = \sum_{ij} \left[ \frac{b_i b_j}{\langle b \rangle^2} \delta(r - r_{ij}) \right] - 4\pi r \rho_0$$

sum over all atoms

distance between  $i$  and  $j$  atoms

average density

# Calculating a PDF from a Model

Calculating a PDF from an atomistic model:

$$G(r) = \sum_{ij} \left[ \frac{b_i b_j}{\langle b \rangle^2} \delta(r - r_{ij}) \right] - 4\pi r \rho_0$$

Peak Width

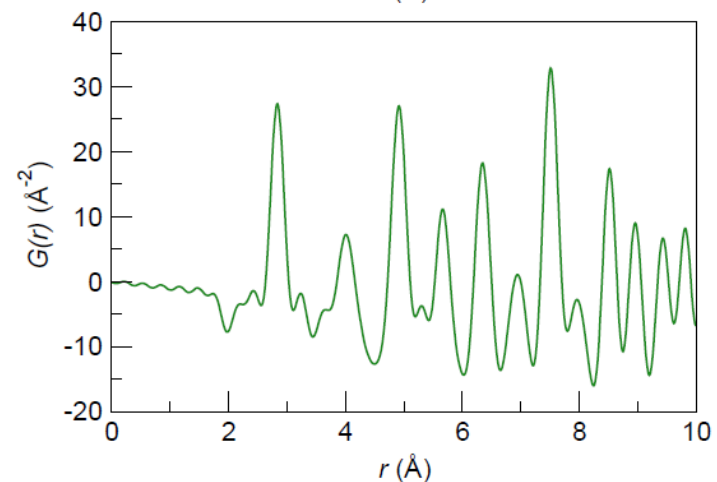
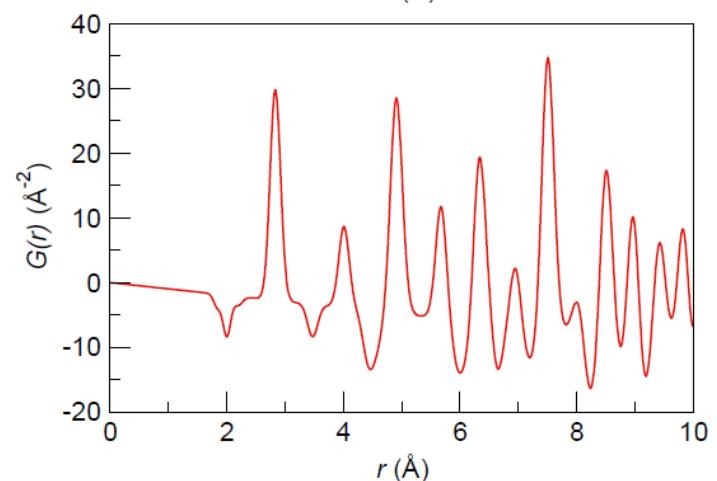
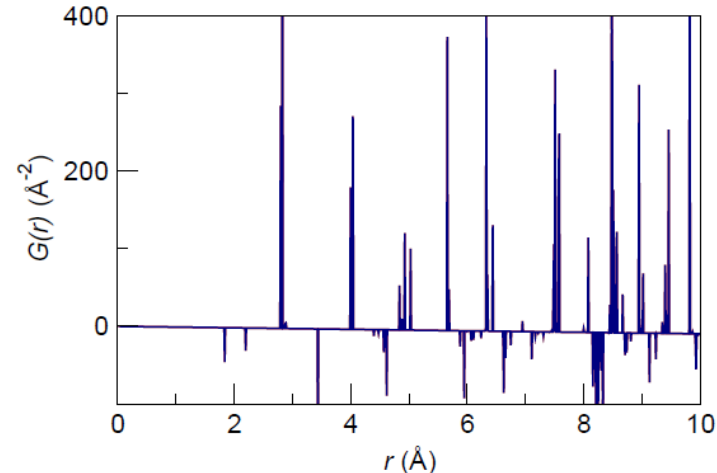
Small model: convolution of  $\delta(r-r_{ij})$  with distribution function (PDFgui & TOPAS v6)

Large model: ensemble average of actual displacements (RMCprofile)

Termination ripples + instrumental dampening

Multiplication with step function in reciprocal space gives convolution with  $\sin(Q_{\max}r)/r$  in real space

...





# Atomic PDF Modeling

## Small Models: Least Squares Refinement

Up to several hundred atoms

'Rietveld'-type parameters: *lattice parameters, atomic positions, displacement parameters, etc.*

Refinements as function of  $r$ -range

## Large Model: Reverse Monte Carlo

20000 + atoms

Fit X-ray and neutron  $F(Q)$ ,  $G(r)$ , Bragg profile

Constraints utilized

Static 3-D model of the structure (a snap-shot)

## Multi-level / Complex Modeling

Refine higher level parameters (not each atom)

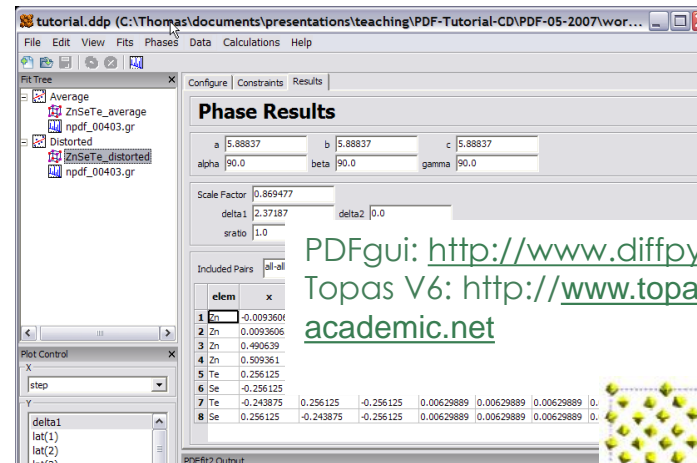
Example nanoparticle: *diameter, layer spacing, stacking fault probability*

Choose minimization scheme

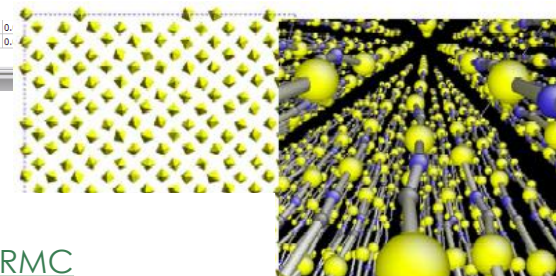
## *ab initio* and force-field based approaches

Density Functional Theory

Molecular Dynamics



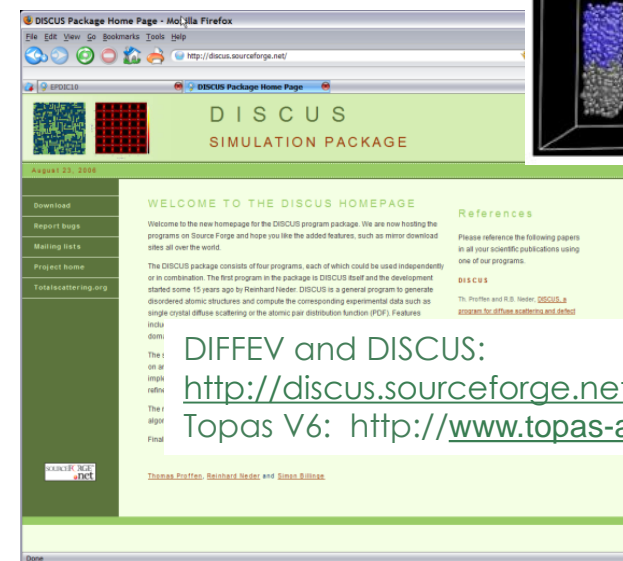
PDFgui: <http://www.diffpy.org/>  
Topas V6: <http://www.topas-academic.net>



RMCprofile:

<http://www.isis.rl.ac.uk/RMC>

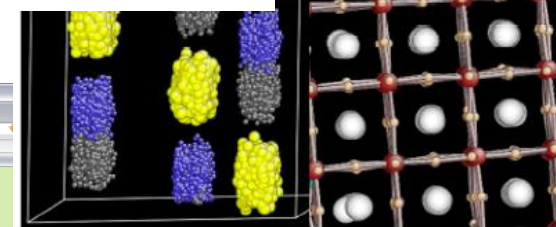
EPSR: <http://disordmat.moonfruit.com/>



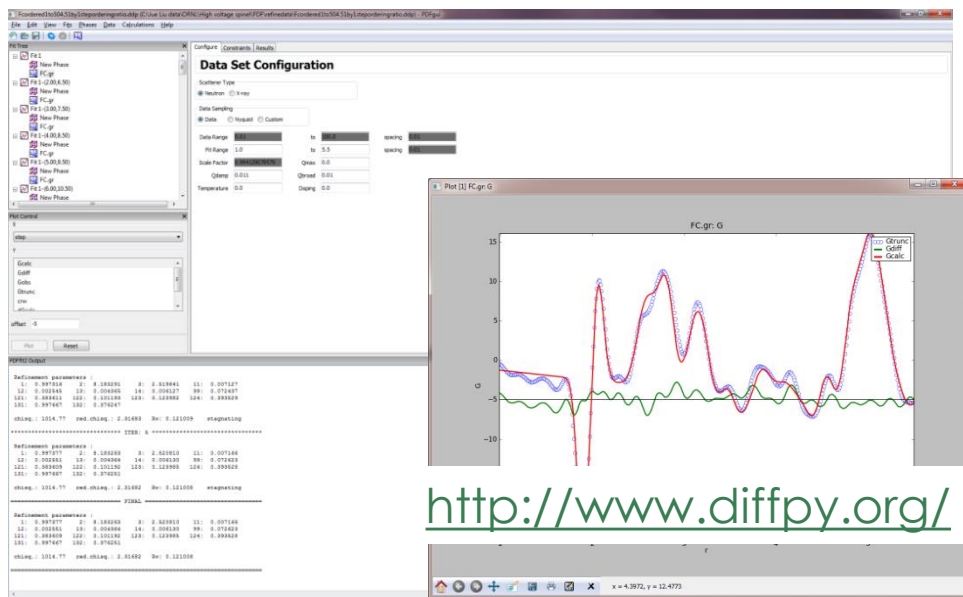
DIFFEV and DISCUS:

<http://discus.sourceforge.net>

Topas V6: <http://www.topas-academic.net>



# Small Box: Software Comparison



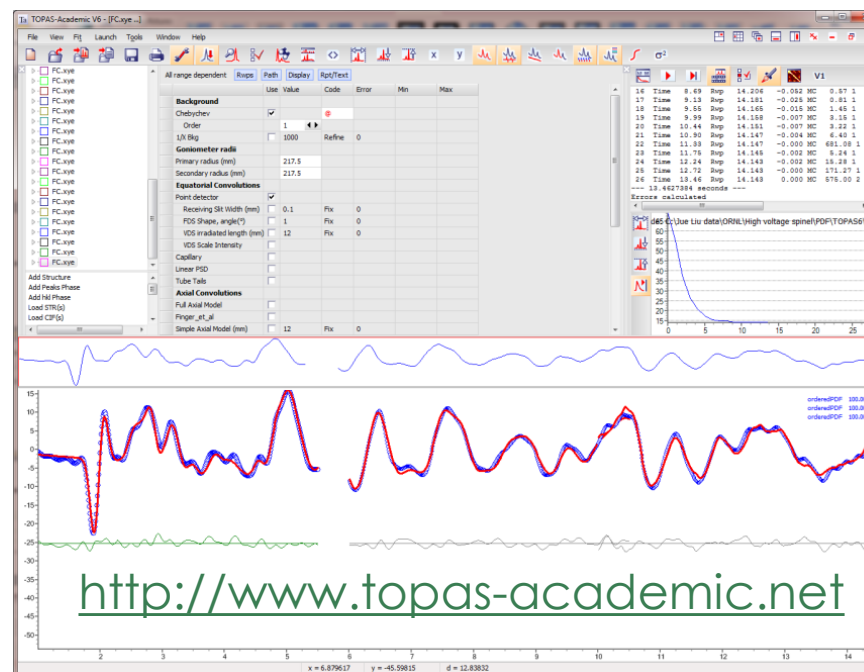
<http://www.diffpy.org/>

## PDFgui

- + Open Source and Free
- + GUI is Simple and User-friendly
- Slow refinement, especially for high- $r$

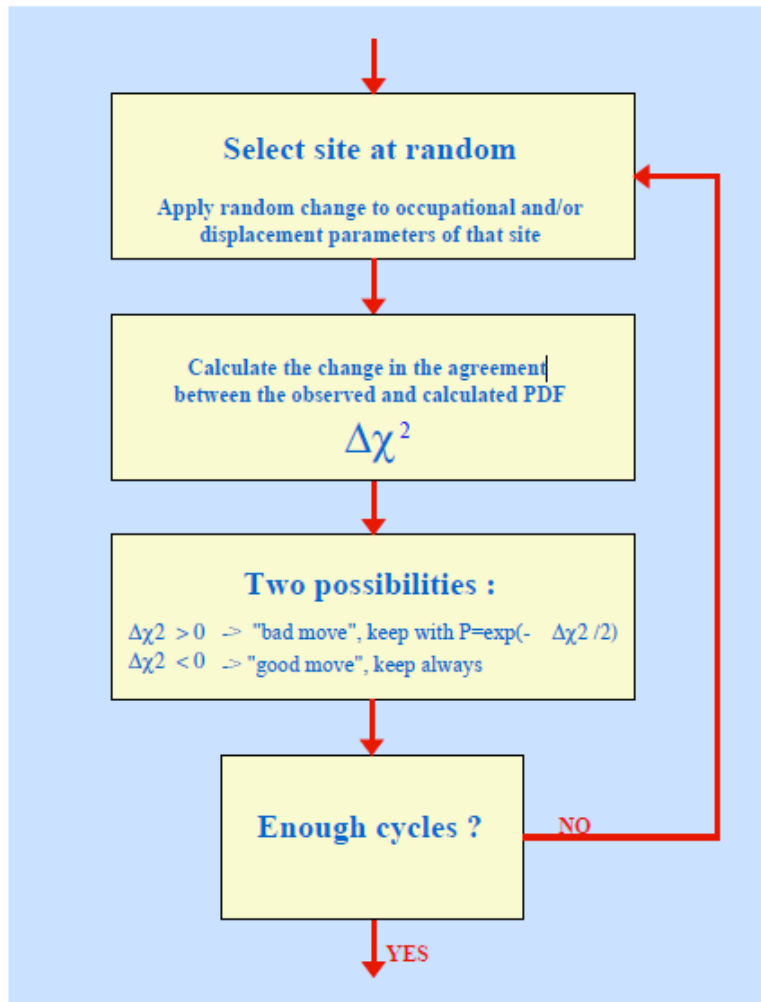
## TOPAS PDF

- + Super Fast
- + Flexible
- + Fit Bragg and PDF together
- Steeper learning curve
- Have to write your own macro



<http://www.topas-academic.net>

# Large Box: Reverse Monte Carlo



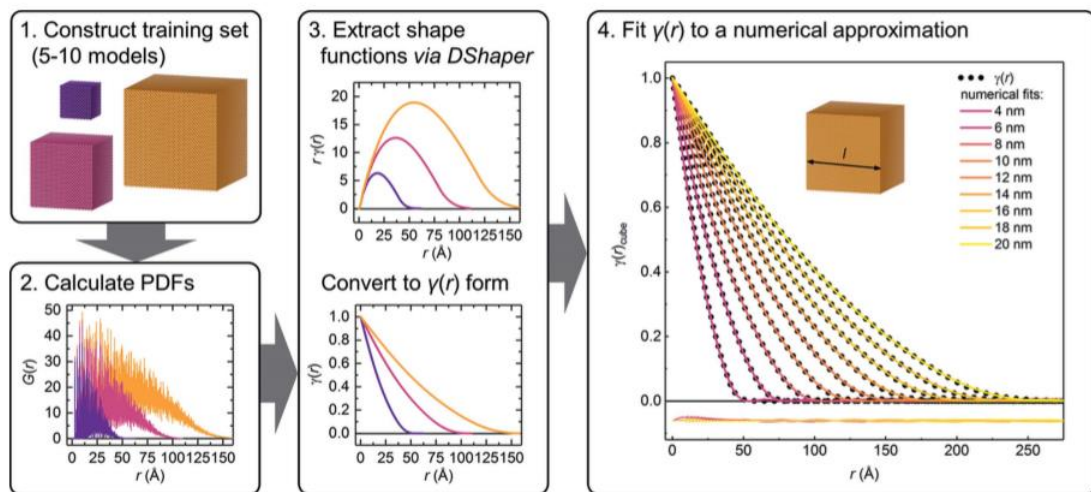
## RMCprofile

- Atomic configurations ~600 to 20000+ atoms
  - Fit both X-ray and neutron  $F(Q)$
  - Fit  $G(r)$
  - Fit Bragg profile (GSAS ToF 1-3)
  - Polyhedral restraints
  - Coordination constraints
  - Closest approach constraints
- **Produce a static 3-D model of the structure (a snap-shot in time)**

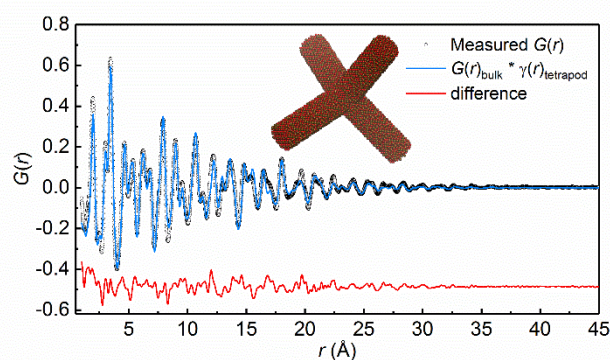
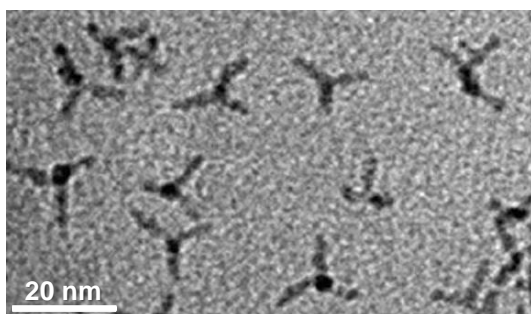
<http://www.isis.rl.ac.uk/RMC>

# Modeling nanoscale morphology in real space

$$G(r) = \frac{2}{\pi} \int_{Q_{min}}^{Q_{max}} Q[S(Q) - 1] \sin(Qr) dQ = 4\pi r[\rho(r) - \rho_0 \gamma_0(r)]$$



- $\gamma_0(r)$  is the particle shape function, it varies significantly from unity for nanomaterials and should be implemented as an  $r$ -dependent function
- Can fit physically-relevant shape parameters, such as a nanocube edge length, nanorod length and diameter, or arm length, width, and arm tip-to-arm tip distance in  $\text{Fe}_2\text{O}_3$  tetrapods (left)
- Options exist in DISCUS, Topas-v6, and Diffpy

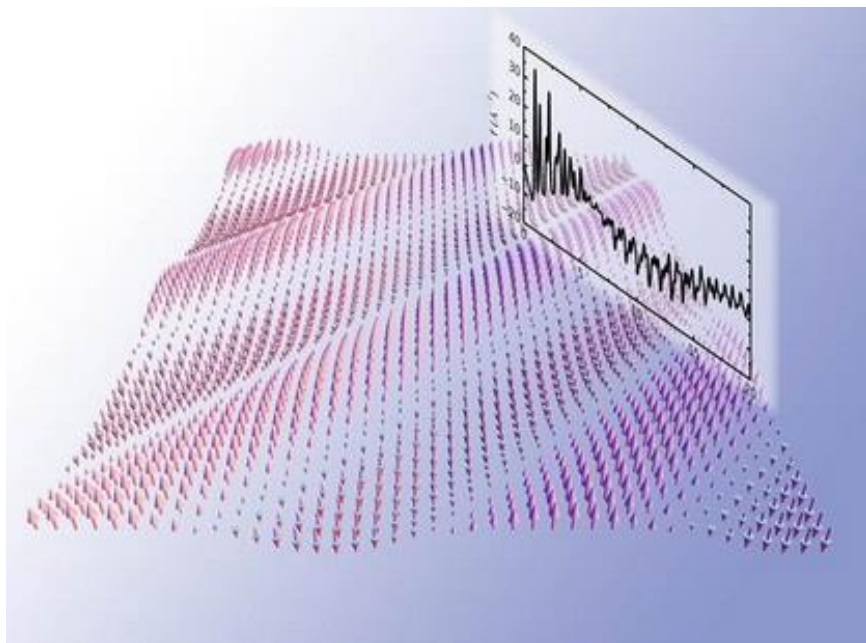


# A Few Emerging Areas

- Magnetic PDF
- Field-dependent PDF
- Dynamic PDF
- 3D PDF
- Thin-Film PDF



# Magnetic PDF: mPDF



## ARTICLE

Received 14 Jul 2016 | Accepted 4 Nov 2016 | Published 20 Dec 2016

DOI: 10.1038/ncomms13842

OPEN

## Emergent order in the kagome Ising magnet $\text{Dy}_3\text{Mg}_2\text{Sb}_3\text{O}_{14}$

Joseph A.M. Paddison<sup>1,2</sup>, Harapan S. Ong<sup>1</sup>, James O. Hamp<sup>1</sup>, Paromita Mukherjee<sup>1</sup>, Xiaojian Bai<sup>2</sup>, Matthew G. Tucker<sup>3,4</sup>, Nicholas P. Butch<sup>5</sup>, Claudio Castellano<sup>1</sup>, Martin Mourigal<sup>2</sup> & S.E. Dutton<sup>1</sup>



Acta Crystallographica Section A  
Foundations and  
Advances  
ISSN 2053-2733

Received 10 October 2013  
Accepted 6 December 2013

advances

## Magnetic pair distribution function analysis of local magnetic correlations

Benjamin A. Frandsen,<sup>a</sup> Xiaohao Yang<sup>b</sup> and Simon J. L. Billinge<sup>b,c\*</sup>

<sup>a</sup>Department of Physics, Columbia University, New York, NY 10027, USA, <sup>b</sup>Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY 10027, USA, and <sup>c</sup>Condensed Matter Physics and Materials Science Department, Brookhaven National Laboratory, Upton, NY 11973, USA. Correspondence e-mail: sb2896@columbia.edu

- Being developed to provide direct access to long-range and short-range magnetic correlations in real space
- **Spin order in diluted magnetic semiconductors, spin-stripe correlations in cuprate superconductors, spin fluctuations in frustrated magnetic systems, etc.**

PRL 116, 197204 (2016)

PHYSICAL REVIEW LETTERS

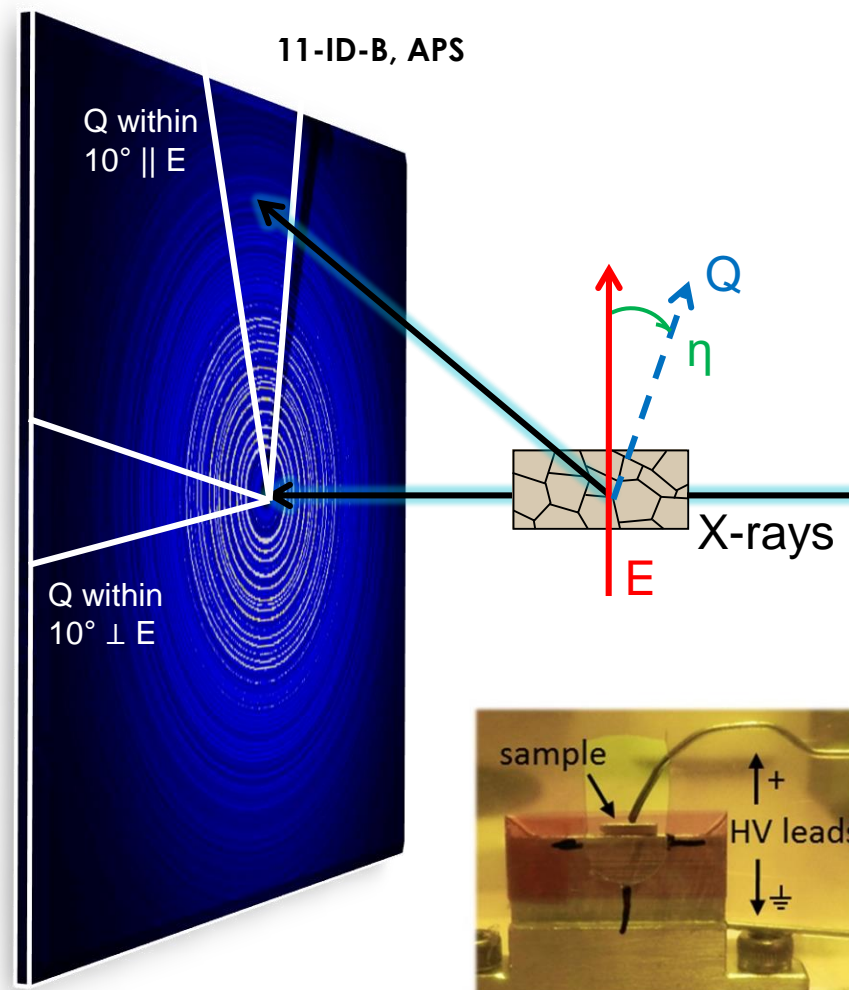
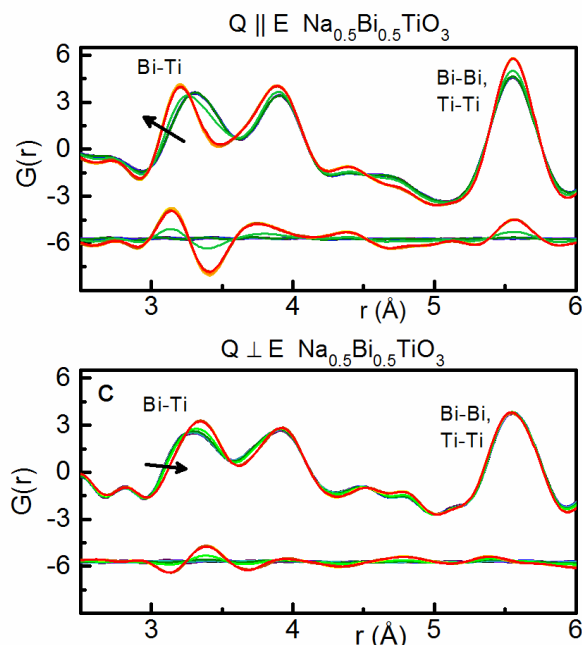
week ending  
13 MAY 2016

## Verification of Anderson Superexchange in MnO via Magnetic Pair Distribution Function Analysis and *ab initio* Theory

Benjamin A. Frandsen,<sup>1</sup> Michela Brunelli,<sup>2</sup> Katharine Page,<sup>3</sup> Yasutomo J. Uemura,<sup>1</sup> Julie B. Staunton,<sup>4</sup> and Simon J.L. Billinge<sup>5,6,\*</sup>

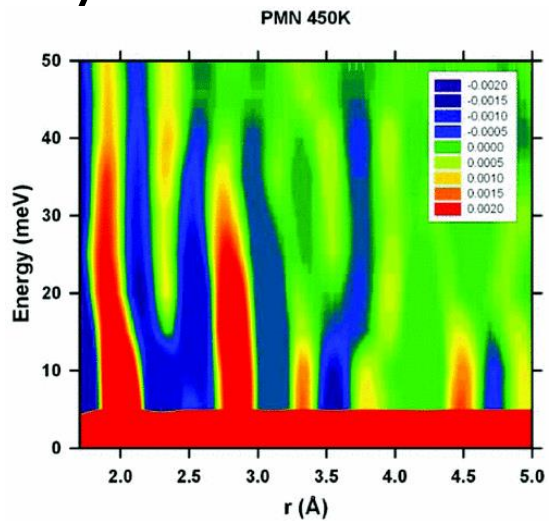
# Field-Dependent PDF

- X-ray total scattering measured while static electric fields (0 to ~4 kV/mm) are applied to  $\text{Na}_{1/2}\text{Bi}_{1/2}\text{TiO}_3$  polycrystalline ceramic samples
- $\text{Bi}^{3+}$  reorientation observed at high electric field



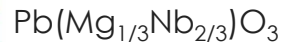
T.-M. Usher, I. Levin, J.E. Daniels, and J.L. Jones, *Scientific Reports* 5, 14678 (2015).  
A. J. Goetzee-Barral et al., *Phys. Rev. B* 96, 014118 (2017).

# Dynamic PDF: DyPDF



T. Egami and W. Dmowski, **Dynamic pair-density function method for neutron and X-ray inelastic scattering**, *Z. Kristallogr.* 227, 233–237 (2012).

W. Dmowski, S. B. Vakhrushev, I.-K. Jeong, M. P. Hehlen, F. Trouw, T. Egami, **Local Lattice Dynamics and the Origin of the Relaxor Ferroelectric Behavior**, *Phys. Rev. Lett.* 100, 137602 (2008).



ARTICLE

Received 28 Jun 2016 | Accepted 17 Mar 2017 | Published 4 May 2017

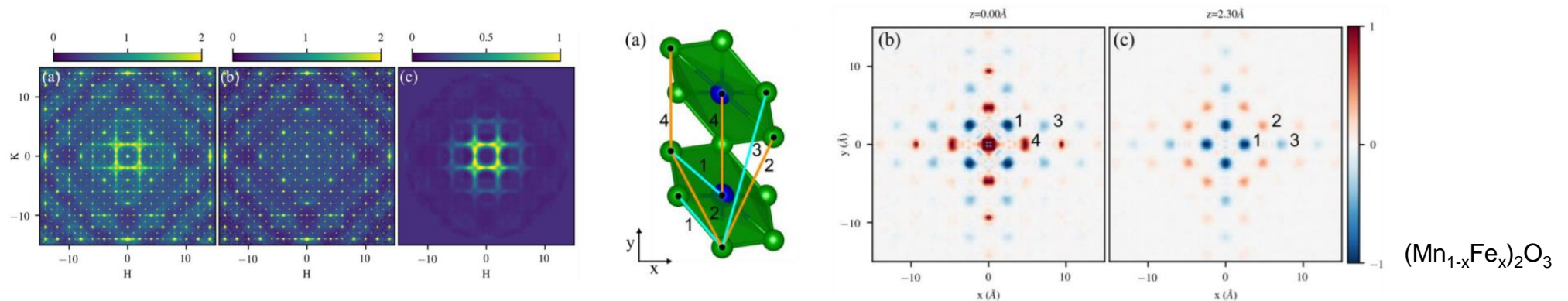
DOI: 10.1038/ncomms15234 OPEN

Observation of dynamic atom-atom correlation in liquid helium in real space

W. Dmowski<sup>1,2</sup>, S.O. Diallo<sup>3</sup>, K. Lakshin<sup>1,2</sup>, G. Ehlers<sup>3</sup>, G. Ferré<sup>4</sup>, J. Boronat<sup>4</sup> & T. Egami<sup>1,2,3,5</sup>

# 3D - PDF

T. Weber and A. Simonov, **The three-dimensional pair distribution function analysis of disordered single crystals: basic concepts**, *Z. Kristallogr.* 227, 238–247 (2012).

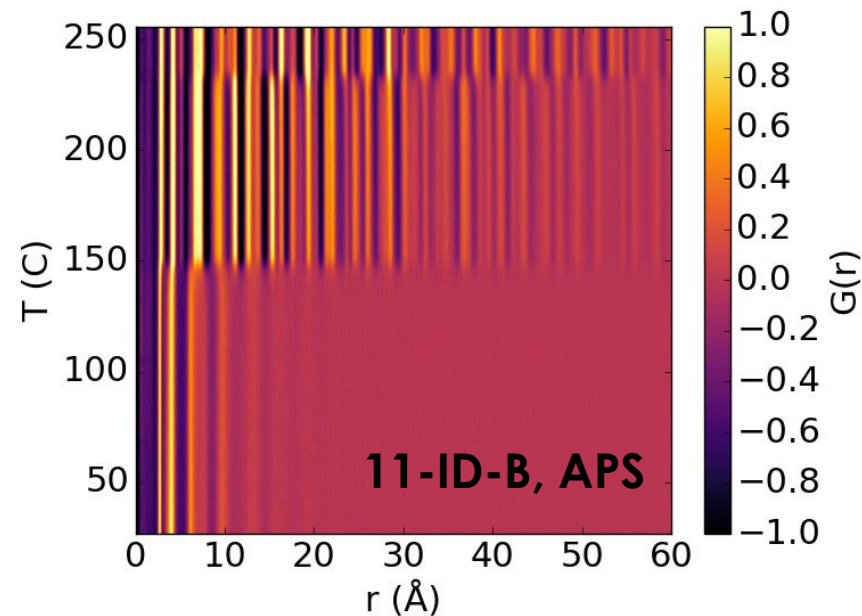
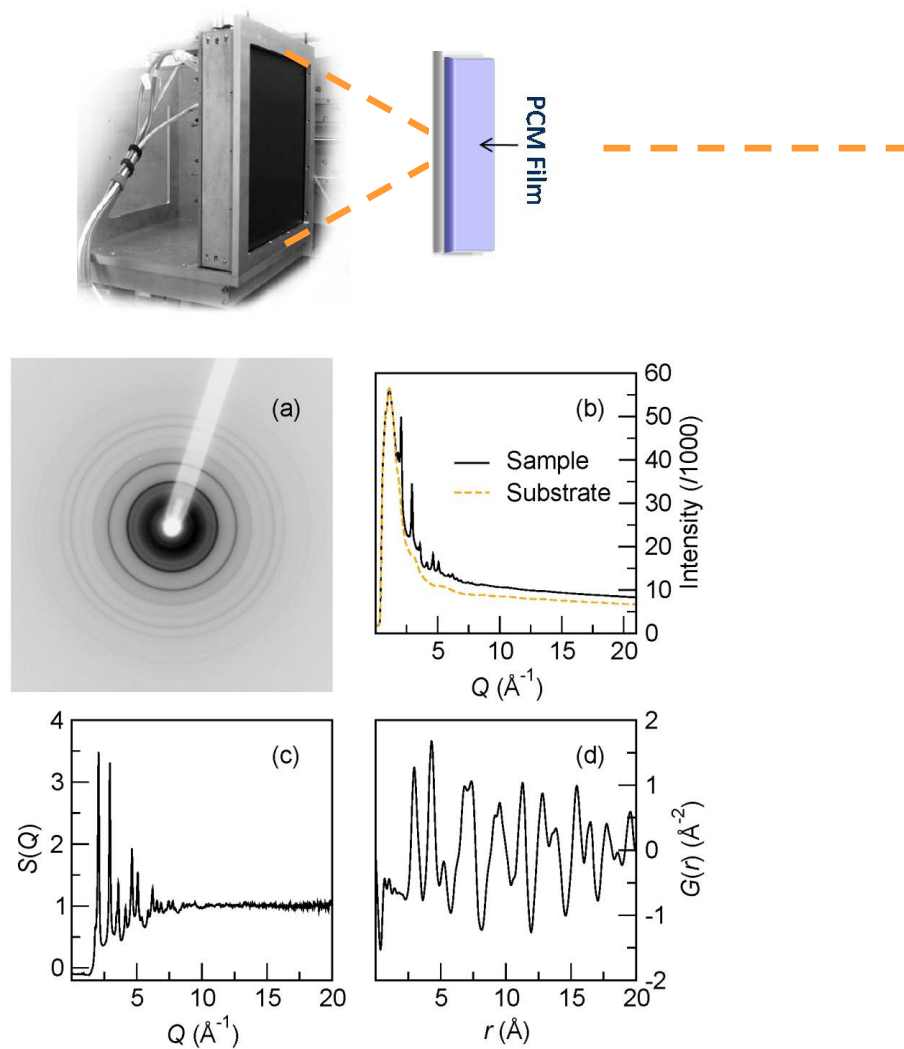


N. Roth, A. F. May, F. Ye, B. C. Chakoumakos, and B. B. Iversen, **Model-free reconstruction of magnetic correlations in frustrated magnets**, *IUCrJ.* 5, 410–416 (2018).



# Thin Film PDF (tfPDF)

K. M. Ø. Jensen, A. B. Blichfeld, S. R. Bauers, S. R. Wood, E. Dooryhée, D. C. Johnson, B. B. Iversen, and S. J. L. Billinge, **Demonstration of thin film pair distribution function analysis (tfPDF) for the study of local structure in amorphous and crystalline thin films**, *IUCrJ*, 2 (2015) 481-489.



Data collected for 1  $\mu\text{m}$  films deposited **on kaptan**, thermally annealed in situ under flowing He to 155°C and measured at ~60 keV in transmission

K. Page, J. K. Baldwin, Th. Proffen, unpublished.

# When Should You Pursue PDF Studies of a Crystalline Material?

- ✓ You have modeled everything you can in reciprocal space
- ✓ You suspect the local structure may differ from the long-range structure

## Why Would You Suspect a Distinct Local Structure?

Maybe...

- You find signatures of disorder through complementary methods
- An average structure model fails to explain observed material properties
- A theoretical study proposes an alternate structure to the one globally observed
- Lots of experience with a materials family or structural archetype

# Some Resources and Programs

## Data Collection

- Neutron: <http://neutronsources.org>
- X-ray: <http://www.lightsources.org>

## Data Extraction

- PDFgetN: <http://pdfgetn.sourceforge.net>
- PDFgetX2/X3: <http://www.pa.msu.edu/cmp/billinge-group/programs/PDFgetX2/> <http://www.diffpy.org/products/pdfgetx3.html>
- Gudrun: <http://disordmat.moonfruit.com/>
- ADDIE: ADvanced DIffraction Environment: coming soon from ORNL!

## Data Modeling

- PDFgui: <http://www.diffpy.org/>
- Topas Academic: <http://www.topas-academic.net>
- RMCprofile: <http://www.isis.rl.ac.uk/RMC>
- DISCUS/DIFFEV: <http://discus.sourceforge.net>
- EPSR: <http://disordmat.moonfruit.com/>

# References & Reviews

S.J.L. Billinge and I. Levin, The Problem with Determining Atomic Structure at the Nanoscale, *Science* 316, 561 (2007). <http://dx.doi.org/10.1126/science.1135080>

T. Egami and S. J. L. Billinge, *Underneath the Bragg peaks: structural analysis of complex materials*, Pergamon Press Elsevier, Oxford, England, 2003.

D. A. Keen, Derivation of commonly used functions for the pair distribution function technique *J. Appl. Cryst.* 34 (2001) 172-177. <http://dx/doi.org/10.1107/S0021889800019993>

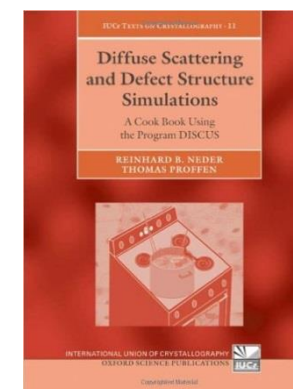
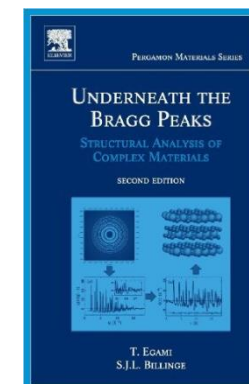
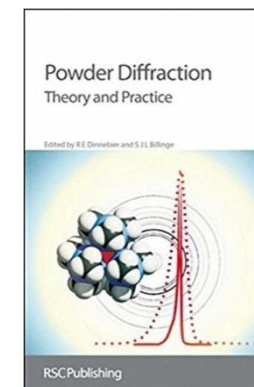
R. Neder and Th. Proffen, *Diffuse Scattering and Defect Structure Simulation*, Oxford University Press, 2008.

M. G. Tucker, M. T. Dove, and D. A. Keen, Application of the reverse Monte Carlo method to crystalline materials, *J. Appl. Cryst.* 34, 630-638 (2001). <http://dx.doi.org/10.1107/S002188980100930X>

D. A. Keen and A. L. Goodwin, The crystallography of correlated disorder, *Nature* 521, 303-309, 2015. <http://dx.doi.org/10.1038/nature14453>

H. Y. Playford, L. R. Owen, I. Levin, and M. G. Tucker, New insights into complex materials using Reverse Monte Carlo modeling, *Annual Review of Materials Research*, 44, 429-449, 2014. <http://dx.doi.org/10.1146/annurev-matsci-071312-121712>

D. Olds, C. N. Saunders, M. Peters, T. Proffen, J. N. Neuefeind, and K. Page, Precise implications on real-space PDF modeling from effects intrinsic to modern time of flight neutron diffractometers, *Acta Cryst.* A74 (2018). <https://doi.org/10.1107/S2053273318003224>



# Summary

**Atomic PDF from total (Bragg and diffuse) scattering data gives access to:**

- Amorphous and nanomaterial structure
- Departure from long range (average structure)
  - Displacements
  - Chemical short-range order
  - Interstitials/vacancies
- Correlation length scale of features (size)
- Structure  $\Leftrightarrow$  property relationships

**Use multiple data sets (e.g. x-ray and neutron data, diffraction and PDF) to characterize complex materials**

**High-resolution instruments open the door to medium-range order investigations**

# Programs/Partnerships for Users

## Open Proposal Call for 2020 Cycle A

- SNS or HFIR's User Program Instruments (diffraction, small angle, reflectometry, spectroscopy, neutron imaging...)
- Next Proposal deadline: **September 2019**

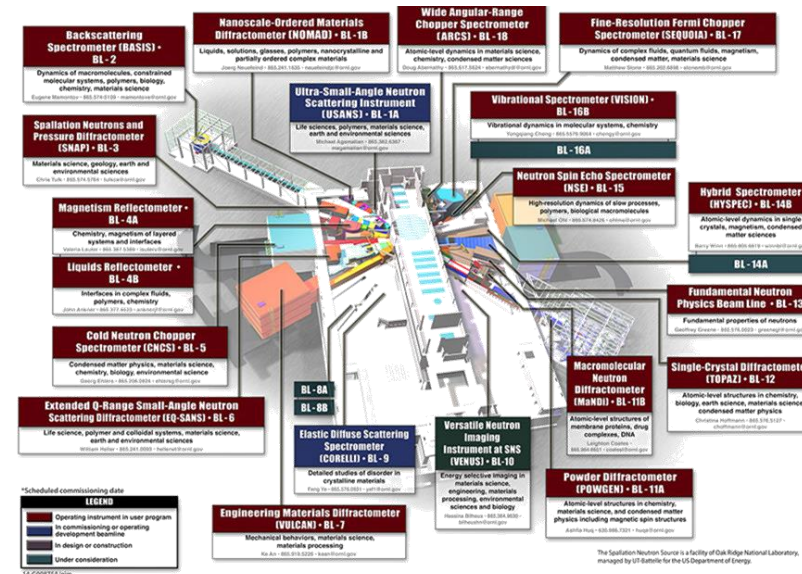
## Mail-in Programs (NOMAD and POWGEN)

- Up to five samples or temperatures (in standard mode)
- Opportunities on most weeks during the run cycle: **Submit Any Time**

## Complimentary x-ray PDF and x-ray Diffraction access

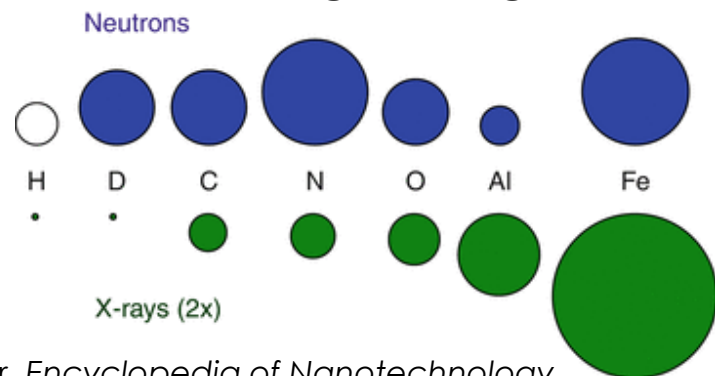
- Measure samples ~3 times per year, and provide reduced data

<http://neutrons.sns.gov>



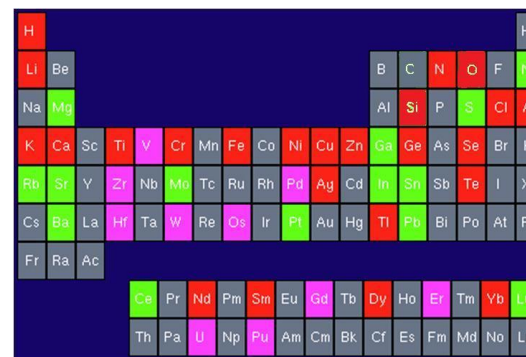
# Neutron Total Scattering

## Light Atom and Neighboring Atom Species



M. Laver, *Encyclopedia of Nanotechnology* (2012), 2437-2450.

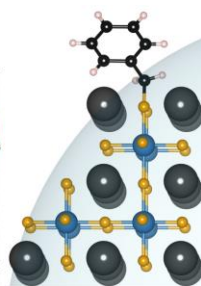
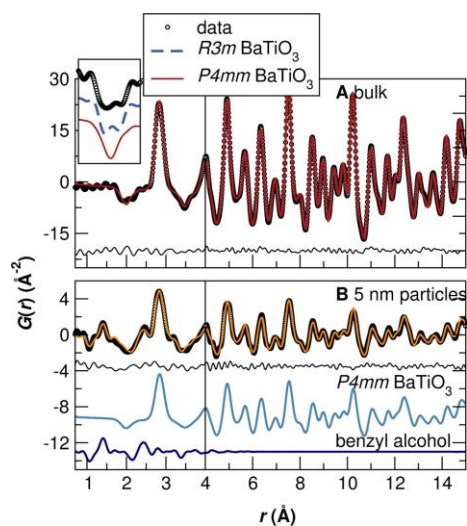
## Isotope Substitution



J. E. Enderby, D.M. North, P. A. Egelstaff,  
**Partial structure factors of liquid Cu-Sn**, *Phil. Mag.* 14 (1966) 131.

Louca, Kwei, Dabrowski, Bukowski,  
*Phys. Rev. B*, (1999) 60, 7558-7564.

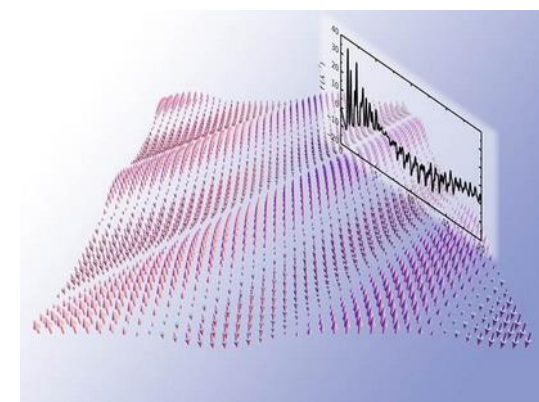
## Surface Species of Nanomaterials



K. Page, Th. Proffen, M. Niederberger, and R. Seshadri,  
**Probing local dipoles and ligand structure in  $BaTiO_3$  nanoparticles**, *Chem. Mater.* 22 (2010), 4386-4391.

## Magnetic Structure

B. Frandsen, X. Yang and S. J. L. Billinge,  
**Magnetic pair distribution function analysis of local magnetic correlations**, *Acta Cryst. A* 70 (2014), 3-11.



## Nondestructive

## Penetration of Sample Environments

# Questions?



[pagekl@ornl.gov](mailto:pagekl@ornl.gov)