SYNCHROTRON RADIATION: PRODUCTION & PROPERTIES

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PROPERTIES OF THE X-RAYS

RADIATION PATTERNS FROM ACCELERATING CHARGES

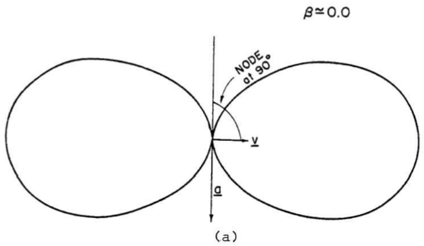
Definitions:

$\beta = v/c$

$\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$

When v << c, ($\beta \approx 0$), the shape of the radiation pattern is a classical dipole pattern.

$\beta = 0$



But as β approaches 1, the shape of the radiation pattern changes; it is more forward directed

$$\tan(\psi)_{lab} = \frac{\sin\psi'}{\gamma\left(\cos\psi' + \beta\right)} = \frac{1}{\gamma\beta} \approx \frac{1}{\gamma}$$

Lorentz transformation

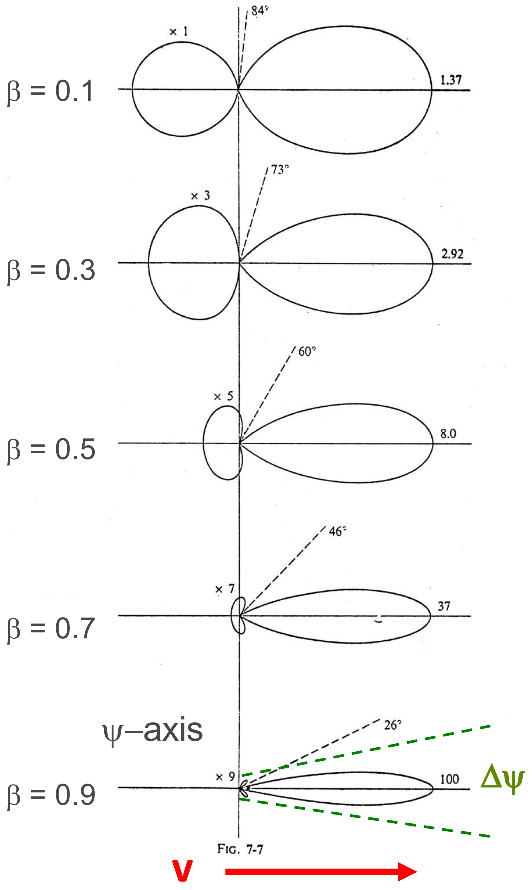
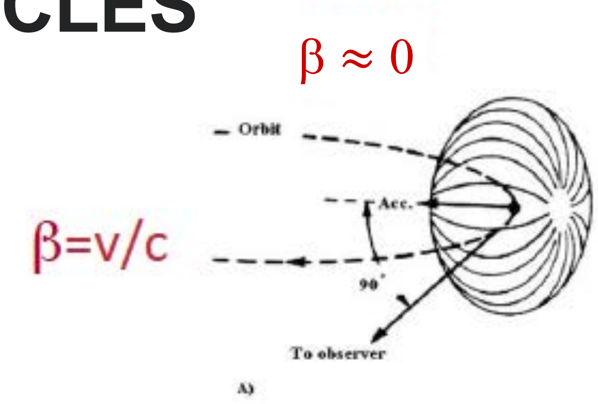
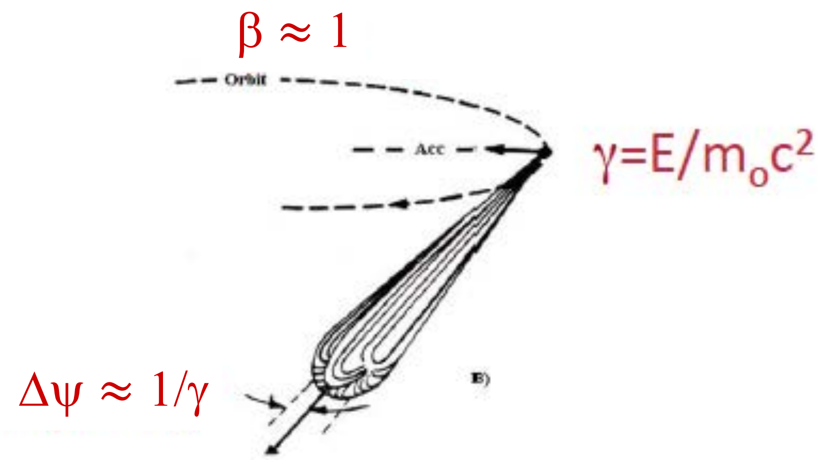


FIG. 7-7

The angular divergence of the radiation (sometimes called the opening angle), Δψ, **is approximately 1/γ.**

RADIATION FROM HIGHLY RELATIVISTIC $(\gamma >> 1)$ PARTICLES





At the APS with E = 7 GeV,

$\gamma = E/m_o c^2 = 7\ \text{GeV}/0.511\ \text{MeV}$

$\gamma = 1.4 \times 10^4$

$1/\gamma = 73 \times 10^{-6}$

See Appendix 1 for more details

The opening angle in both the horizontal and vertical directions is given approximately by:

$$\Delta\psi_{\text{vert}} = \Delta\psi_{\text{hor}} \approx 1/\gamma,$$

when $\beta \approx 1$. (@ APS $\beta \approx 1 - 2 \times 10^{-9}$)

Relativistic velocities are good!!

- radiation forward directed
- radiated power $\propto E^4$

ANATOMY OF SYNCHROTRON RADIATION FACILITY

ADVANCED PHOTON SOURCE

BEAM ACCELERATION & STORAGE SYSTEM

(A) ELECTRON GUN

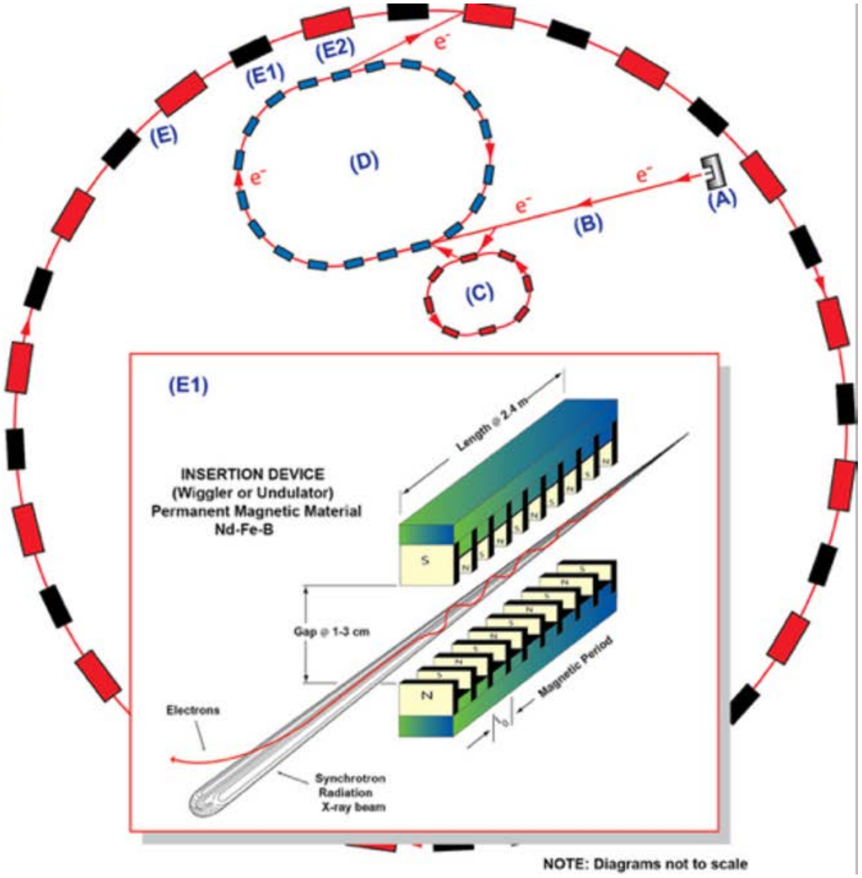
(B) ELECTRON LINEAR ACCELERATOR
Output energy: 375 MeV

(C) PARTICLE ACCUMULATOR RING

(D) BOOSTER SYNCHROTRON
Nominal extraction energy: 7.0 GeV

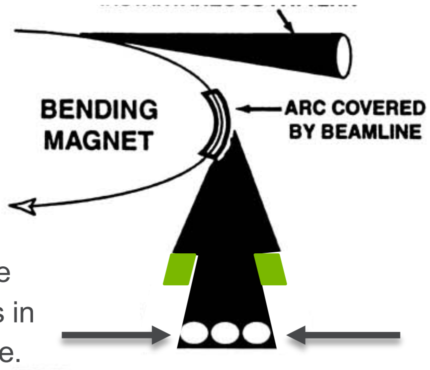
(E) STORAGE RING
Nominal energy: 7.0 GeV

(E1) INSERTION DEVICE
(E2) BENDING MAGNET

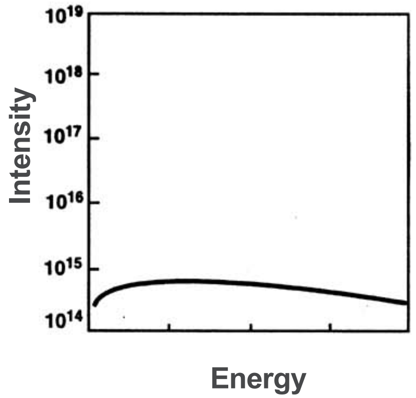


NOTE: Diagrams not to scale

BEND MAGNET X-RAY RADIATION PROPERTIES



Horizontal opening angle determined by apertures in the front-end or beamline.



You get this for “free” since you need the bending magnets to keep the electrons orbiting in the storage ring.

Bend Magnet Radiation

Spectrum characterized by the critical energy:

E_c[keV] = 0.066 B[kG] E^2 [GeV] ($E_c \approx$ 20 keV @ APS)

Recall: λ[Å] = 12.4/E[keV] so 20 keV is 0.62Å

Vertical opening angle ($\Delta\psi_v$) is $1/\gamma$. *At the APS:*

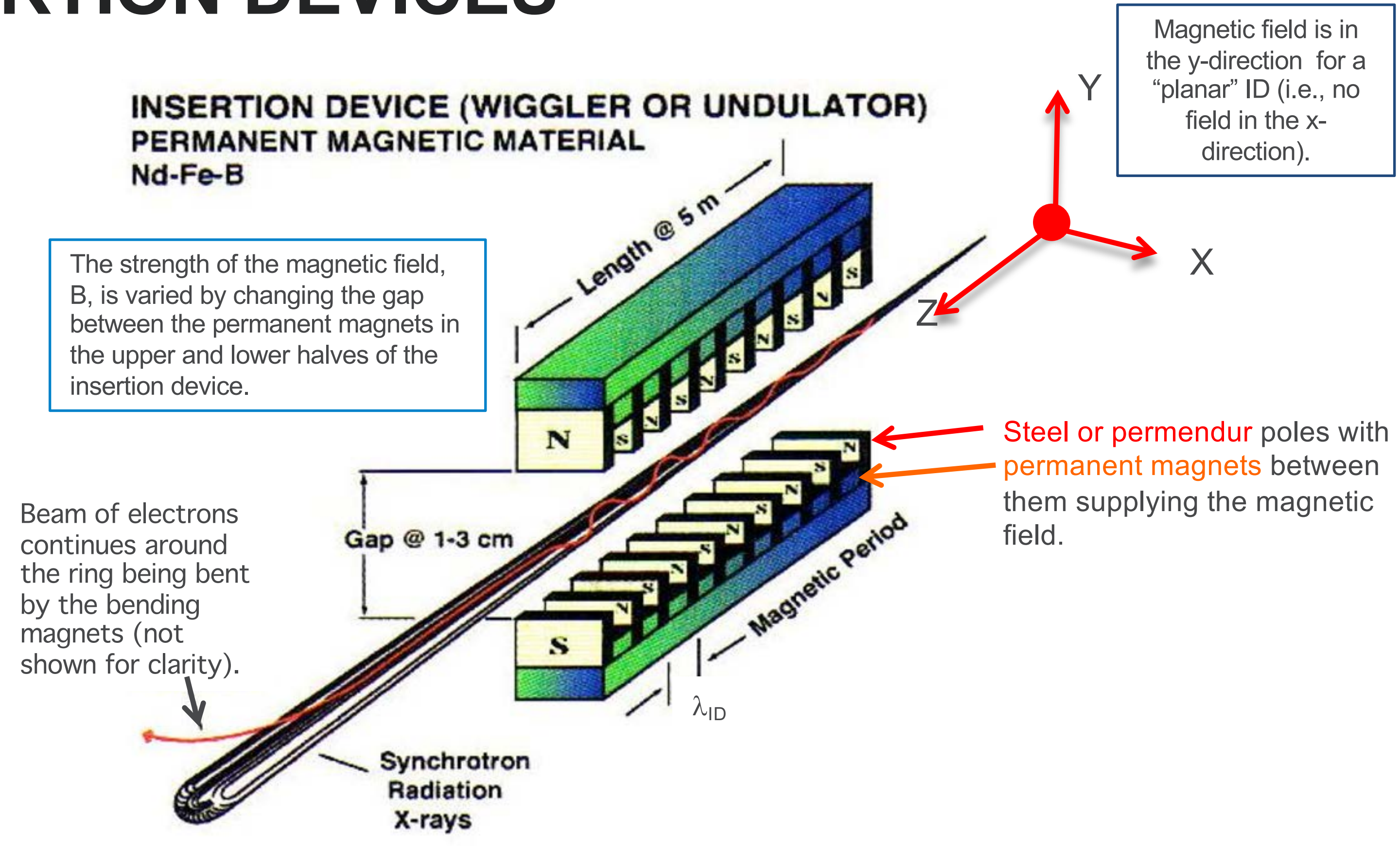
$\gamma = 1.4 \times 10^4$

so $1/\gamma = 73 \times 10^{-6}$ radians

See Appendix 2 for more details

PLANAR INSERTION DEVICES

- Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.
- “Planar” refers to the magnetic field being in one direction (in this case vertical or y-direction).
- IDs can have fields in both the vertical and horizontal directions to produce circularly polarized x-rays and for other applications.



Magnetic field is in the y-direction for a “planar” ID (i.e., no field in the x-direction).

The strength of the magnetic field, B, is varied by changing the gap between the permanent magnets in the upper and lower halves of the insertion device.

Beam of electrons continues around the ring being bent by the bending magnets (not shown for clarity).

Steel or permendur poles with permanent magnets between them supplying the magnetic field.

CHARACTERIZING INSERTION DEVICES

- IDs are characterized by the so-called field index or deflection parameter, K

$$K = eB_o\lambda_{ID}/2\pi m_o c = 0.0934\ \lambda_{ID}[\mathrm{cm}]\ B_o[\mathrm{kG}]$$

Important thing here is K is proportional to mag field, B.

where λ_{ID} **is the magnetic period** of the insertion device and $\mathbf{B_o}$ **the peak magnetic field**. (The length of the insertion device, L, is equal to the number of periods, N, times the length of the period, i.e., $L = N\lambda_{ID}$.)

- The maximum deflection angle of the particle beam, θ_{max}, is given by:

$$\theta_{max} = \pm(K/\gamma)$$

and the amplitude of the oscillation of the particles, x_{max}, by:

$$x_{max} = (K/\gamma)(\lambda_{ID}/2\pi)$$

APS Undulator A has a period of 3.3 cm and operates with $K \approx 1$, therefore:

$$\theta_{max} \approx 1/\gamma$$

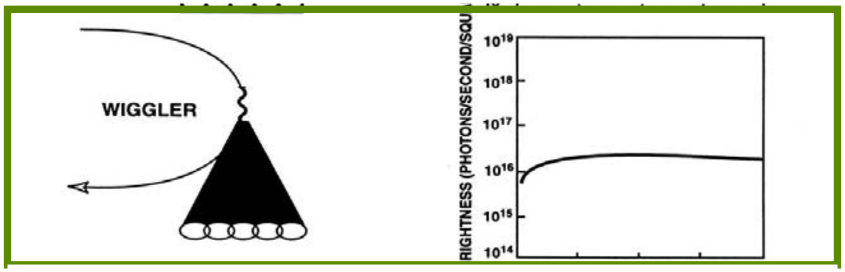
$$x_{max} \approx 0.38 \text{ microns.}$$

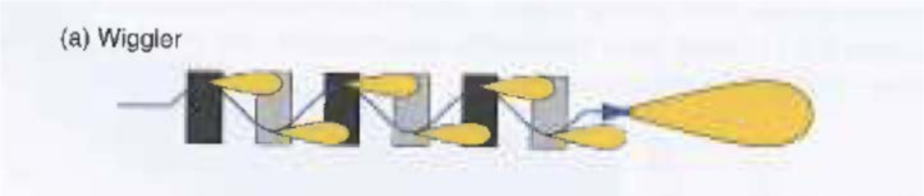
See Appendix 3 for more details

WIGGLER RADIATION PATTERN & SPECTRUM

Horizontal angle given by $\theta_H = (K/\gamma)$

Radiation spectrum looks like 2N dipole sources (N = number of periods)





Top View

Wiggler Radiation (K>>1)

- $\theta_{max} = (K/\gamma) >> 1/\gamma$, i.e. the angular deflection of the particle beam is much greater than the natural opening angle of the radiation ($1/\gamma$).
- Spectrum characterized by the critical energy (same formula as the bending magnet critical energy).
- **Presently, there are NO planar wigglers installed at the APS because they lack the brightness of undulators. (see next slides)**

$\theta_{max} = (K/\gamma) >> 1/\gamma$,

UNDULATORS

Undulators (K ≈ 1)

- If the magnetic field is carefully designed, this overlap can cause interference effects in the spectral distribution.
- On-axis ($\psi_V = \psi_H = 0$), the constructive interference occurs at a particular x-ray wavelength ($\lambda_n^{x\text{-ray}}$) and its odd harmonics, i.e. n = 1,3, 5… when:

The wavelength where constructive interference occurs →

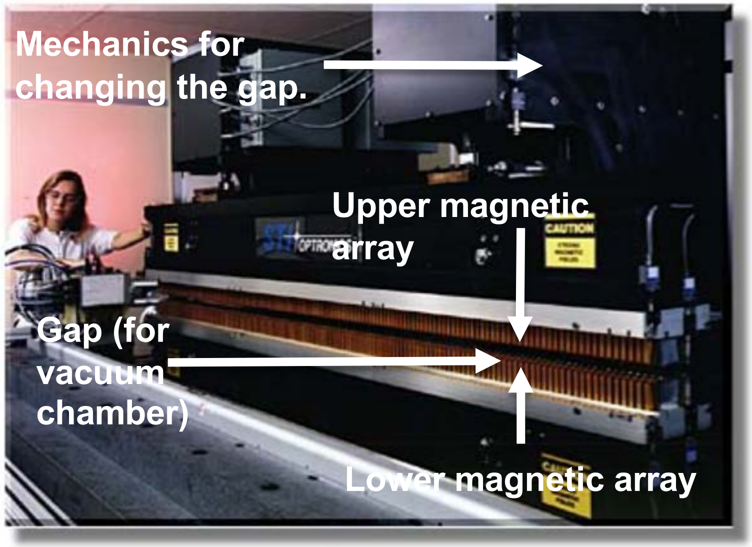
$$\lambda_n^{x\text{-ray}} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

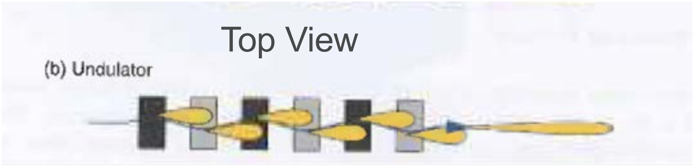
n is the harmonic number – only odd harmonics are generated

K is proportional to the magnetic field strength

- The wavelength, $\lambda_n^{x\text{-ray}}$, where constructive interference occurs, can be adjusted by varying the magnet field, B, of the undulator since $K = 0.093\,\lambda_{ID}[\text{cm}]\,B_o[\text{kG}]$.
- Since most undulators are made with permanent magnets, the B-field is changed by changing the gap between the magnets.

APS 2.4 m long Undulator A (λ_{ID} = 3.3 cm) Permanent magnets





$$\theta_{max} = (K/\gamma) \approx 1/\gamma,$$

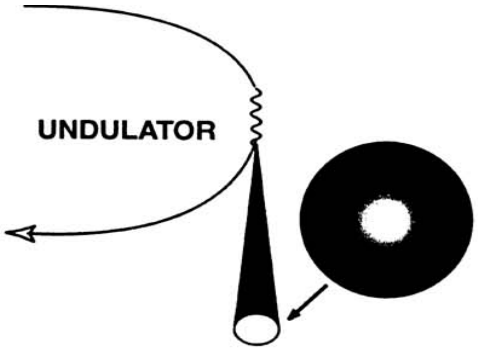
UNDULATOR RADIATION SPECTRA

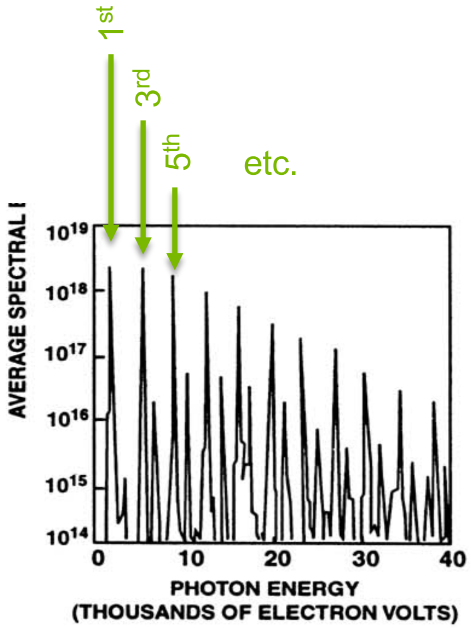
Undulator Radiation (K ≈1)

- The spectrum from an undulator is spikey, but peaks are tunable in energy by varying K since $K \propto B[\mathrm{kG}]\,\lambda_{ID}[\mathrm{cm}]$.
- At the fundamental (1st harmonic or n=1), the horizontal and vertical opening angles of the radiation is given by:

$$\Delta\psi_V = \Delta\psi_H \approx (1/\gamma)\,[1/N]^{1/2}$$

where N = number of periods [typically 100 or so]], **therefore the angular divergence of the undulator first harmonic is ≈ one tenth that from a bending magnet source – a few microradians.**





$$\lambda_n^{x\text{-}ray} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

To get the opening angle of the emitted radiation **observed by the experimenter**, you need to consider the angular properties of the emitting electrons (more later on!), not just the properties of the radiation itself.

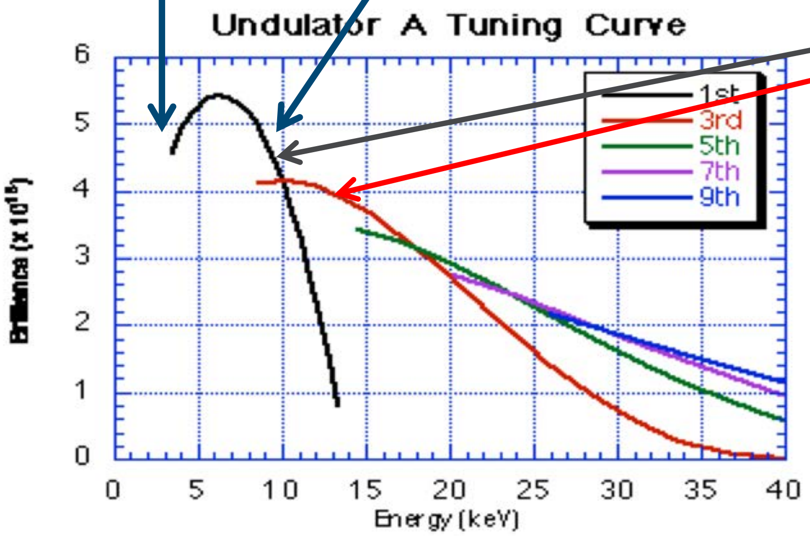
TUNING THE PEAKS OF UNDULATOR RADIATION

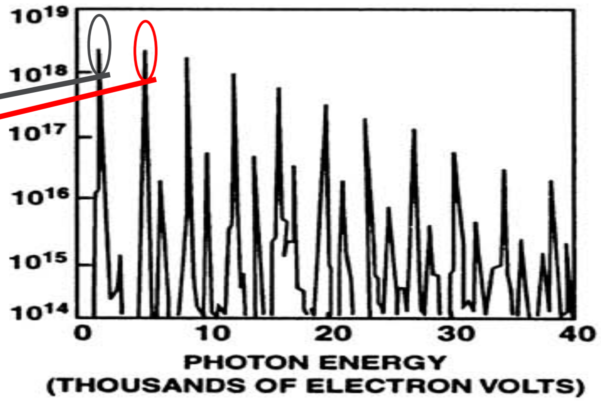
- What is typically shown for the output of an undulator is not the spectrum at one gap setting but rather the energy range that can be covered over the available from the minimum gap.

The lowest x-ray energy (highest magnetic field) is usually fixed by the minimum magnet gap (determined by the vacuum chamber).

As the gap is opened, the magnetic field gets smaller and the constructive interference occurs at higher x-ray energy. But when the magnetic field decreases so does the intensity so that limits the useful high energy range.

Here is the **spectrum generated by an undulator for a particular gap or K value.**





Plots like this that show the energy range that can be covered by an undulator are often called **"tuning curves".**

THE DIFFRACTION LIMIT OF RADIATION

- Just as the Uncertainty Principle sets a lower limit for the product of the size Δx and momentum (Δp), this relationship can be re-written in terms of size Δx and angular divergence $\Delta x'$ for radiation.

$$\sigma_x \sigma'_x = \Delta x\, \Delta x' \geq \lambda / 4\pi \quad \text{and} \quad \sigma_y \sigma'_y = \Delta y\, \Delta y' \geq \lambda / 4\pi$$

primes are used to denote angle coordinates

- The products of source size and source divergence in the x or y plane is proportional to what is called the **emittance** (in optics sometimes called etendue).
- So real-world physical light sources exhibit a **finite emittance due to diffraction effects.**
- The minimum emittance at 1 Å is:

$$1\text{Å} / 4\pi = 10^{-10} \text{ meters} / 4\pi \approx 8 \times 10^{-12} \text{ m} \quad \text{or}$$

8 picometers – radian (radians are dimensionless)

See Appendix 4 for more details

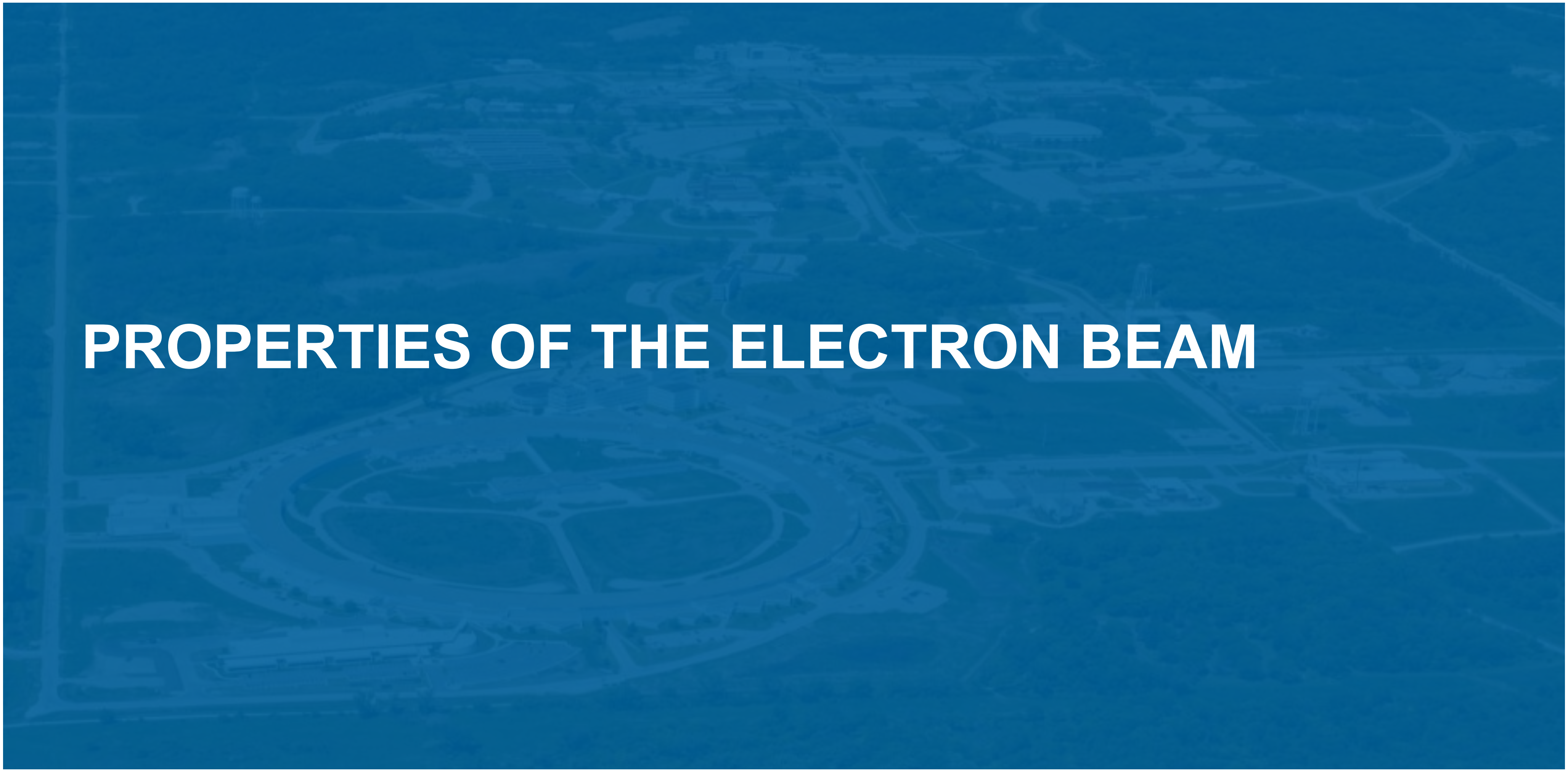
APS Undulator A has a length of 2.4 meters. For 1 Å radiation the natural opening angle is:

$$\sigma'_r = 1/\gamma\, (1/N)^{1/2}$$

$$\sigma'_r = [\lambda/2L]^{1/2} = 4.5\text{x}10^{-6} \text{ rad}$$

The corresponding source size of the radiation is:

$$\sigma_r = [\lambda L / 8\pi^2]^{1/2} = 1.7 \text{ microns.}$$



PROPERTIES OF THE ELECTRON BEAM

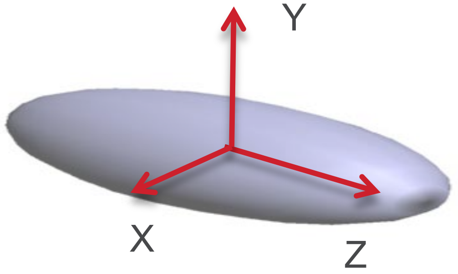
PULSE DURATION & TIME STRUCTURE

- Because the electrons are radiating x-rays, they are constantly losing energy, and so to restore the energy loss on each revolution, radio-frequency (RF) resonant cavities are installed in the storage ring to replenish the radiative energy losses.

- Particles are grouped together by the action of the radio-frequency (RF) cavities into bunches. At APS:
  - 1104 m circumference (3.68 microsecond period)
  - there are 1296 evenly spaced "RF buckets" (stable orbit positions") around the ring
  - the bunch length of the electron packet is about 3 cm in length, corresponding to a **pulse duration** of about 100 psec

- Details of the **time structure** depends on the fill pattern, i.e. which RF buckets have electrons in them.

electron packet in the storage ring

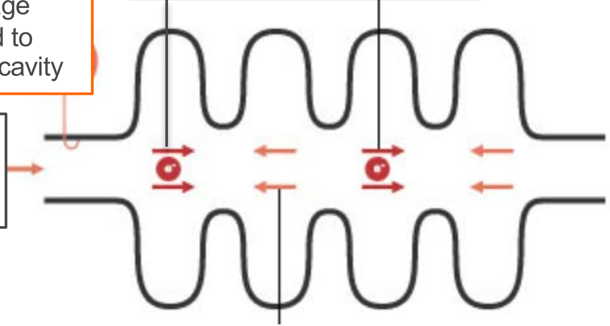


z is in the direction of travel

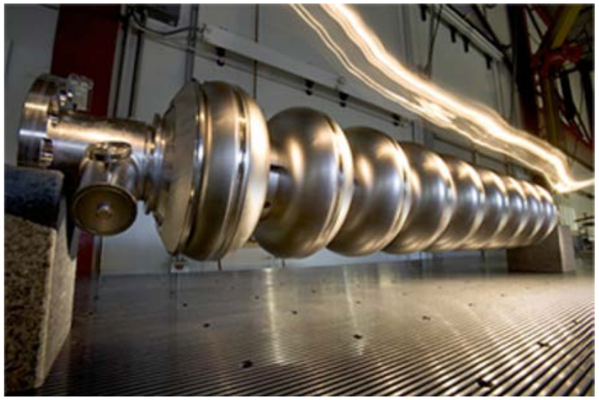
Electrons arrive at the right time to feel the correct E-field to gain the lost energy

rf voltage applied to resonant cavity

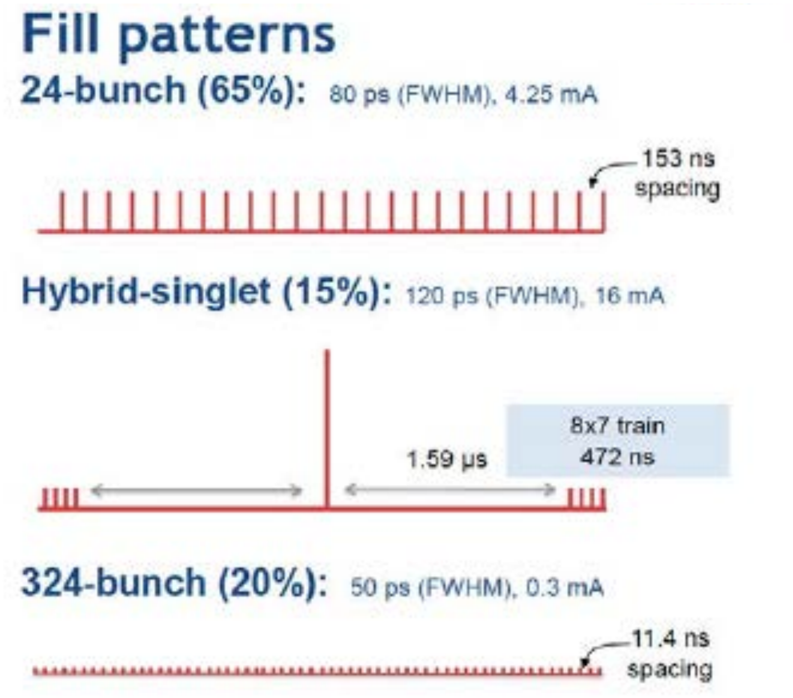
Packets of electrons in the storage ring



A time-varying, standing wave (electric field) is set up in the resonant rf cavity



APS FILL PATTERNS



- The time structure is determined by which of the rf buckets are filled with electrons.

24 equally spaced bunches (about 4 mA/bunch)

- compromise between quasi-continuous source and pulsed source

1 + 8x7 (12-14 mA in one bunch and the rest of the 100 mA distributed in 7 trains of 8 closely spaced bunches)

- timing experiments

324 equally spaced bunches (about 0.3 mA/bunch)

- approximates a continuous source

TRANSVERSE (X & Y) ELECTRON BEAM PROPERTIES

- Although the flux from undulators can be determined without knowledge of the source size and divergence, one very important characteristic of the x-ray beam, namely **brightness,** requires a more detailed knowledge of the particle beam's size and divergence.

Brightness has units of: photons/sec/0.1% BW/source area/source solid angle

$$\text{Flux}/4\pi^2\, \Sigma_h\, \Sigma_v\, \Sigma_h'\, \Sigma_v'$$

this is the monochromaticity of the beam

where Σ_i (Σ_i') is the **effective** one sigma value of the source size (divergence) in the i^th^ direction. If Gaussian distributions are assumed for both the electron beam and the radiation itself, the resultant source size and divergence is the quadrature sum of the two components, namely:

We just determined σ and σ' for undulator radiation

Now we need σ and σ' for the electron beam

$$\Sigma_i = [\sigma_r^2 + \sigma_i^2]^{1/2} \quad \text{and} \quad \Sigma_i' = [\sigma_r'^2 + \sigma_i'^2]^{1/2}.$$

primes indicate angle

radiation beam size

electron beam size (i= x or y)

radiation beam divergence

electron beam divergence (i= x or y)

- The goal of next generation sources is to reduce the electron beam size and divergence to get brighter beams of x-rays.

TRANSVERSE PROPERTIES OF PARTICLE BEAMS

- When a bunch of electrons are injected into a storage ring, the ensemble of electrons rattles around until it reached its equilibrium values, in both the transverse (perpendicular to propagation) and longitudinal (z) directions.

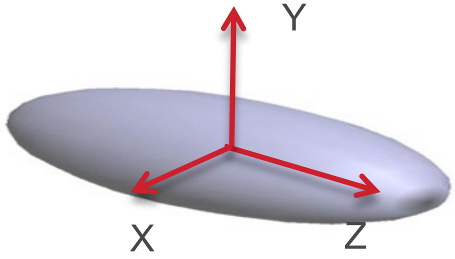
- Here we are interested in the ***transverse*** (x and y) properties.

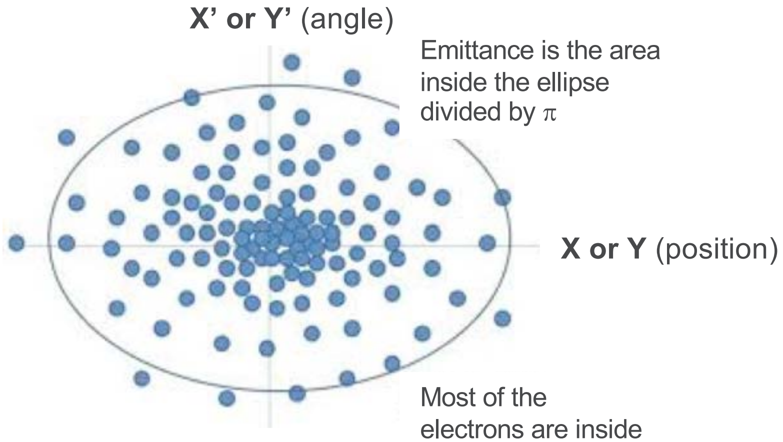
- Accelerator physicists describe the electron beam in terms of its horizontal (x-plane) and vertical (y-plane) position and divergence or phase-space.

- This definition of phase space has as the two conjugate variables

  X and X' (or Y and Y').

- The product of X and X' (or Y and Y') is proportional to what is called the horizontal (or vertical) emittance of the electron beam.





Emittance is the area inside the ellipse divided by π

Most of the electrons are inside the circle

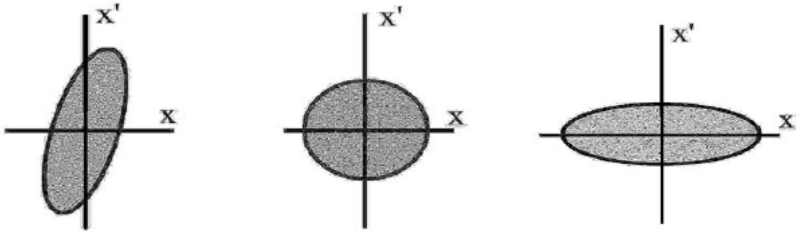
Remember: this plot is not the beam cross-section, but a phase space representation!!!

ELECTRON BEAM PHASE SPACE AND EMITTANCE

- There is a separate horizontal emittance (ε_x) and vertical emittance (ε_y). Typically in today's storage rings:

  ε_y is 1% to 10% of ε_x (the percentage is called the coupling)

- The emittance is a constant of the storage ring, although one can trade off electron beam size for divergence as long as the area of the phase-space remains constant. So you need to know the electron beam source size and divergence at the spot in the storage ring where the undulator is located and add those values in quadrature with source size and divergence of the radiation.



This is a consequence of Liouville's Theorem – the conditions for which are satisfied in most accelerators.

@ APS:

ε_H = *3 x 10^{-9} m-rad or* ***3000 pm-rad***

ε_V = *0.03 x 10^{-9} m-rad or* ***30 pm-rad***

Recall: ***8 pm – rad*** is the diffraction limited emittance @ 1 Å

DIFFRACTION LIMITED SOURCES AND COHERENCE

- If both the horizontal and vertical emittance of the beam were small compared to the radiation emittance, then the x-rays that are emitted would **be fully transversely coherent**.

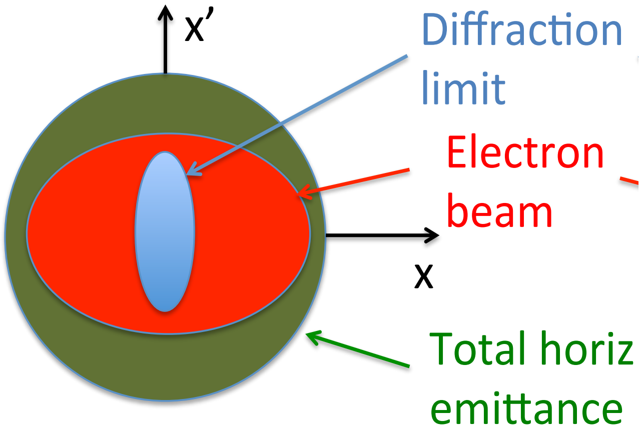
- But at the APS, it is more like the case shown below where the electron beam emittance is dominant.

- Hence the APS is a partially coherent source at 1 Å.

- Partially coherent sources are sometimes characterized by the coherent fraction, the fraction of the x-ray flux that is coherent.

  Coherent fraction = ratio of diffraction-limited emittance to actual beam emittance

- For the APS at 1Å, the coherent fraction is ≈ 10^{-3}. This value is marginally good enough for coherence-based experiments so the trend is to try to reduce the particle beam emittance to increase coherence.



THE DRIVE FOR MORE COHERENT SOURCES

- For 1Å (12 keV) x-rays → 8 picometers – radian for fully coherent beam. APS operates with:

$$\varepsilon_H = 3 \times 10^{-9} \text{ m-rad or } 3000 \text{ picometer-radian}$$

$$\varepsilon_V = 0.03 \times 10^{-9} \text{ m-rad or } 30 \text{ picometer-radian}$$

- At the APS, the present magnets that make of the storage ring lattice limit how low the emittance can be.

- Presently there are two approaches to more coherent x-ray sources:
  - Storage rings Multi-bend Achromat (MBA) magnet structures
  - LINACs that satisfy the conditions for a free electron lasers (FELs)

NEXT GENERATION LIGHT SOURCES

Argonne
NATIONAL LABORATORY

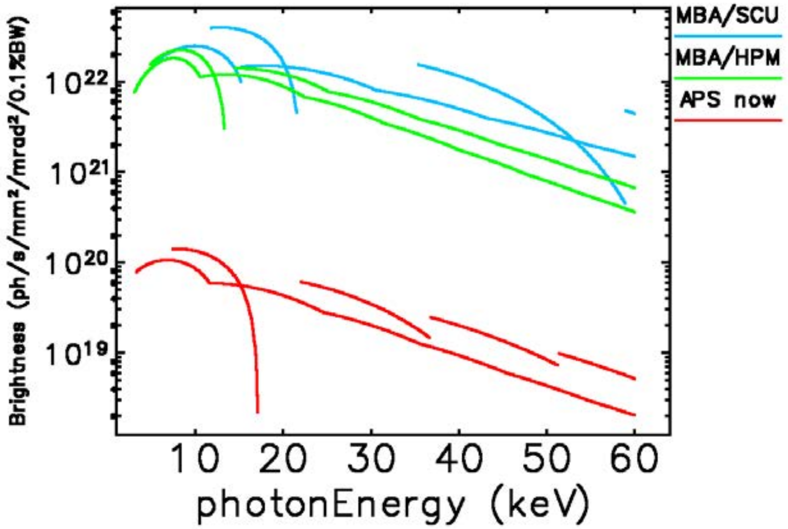
A MULTI-BEND ACHROMAT (MBA) LATTICE AT APS

$$\varepsilon_x = C_L \frac{E^2}{N_D^3}$$

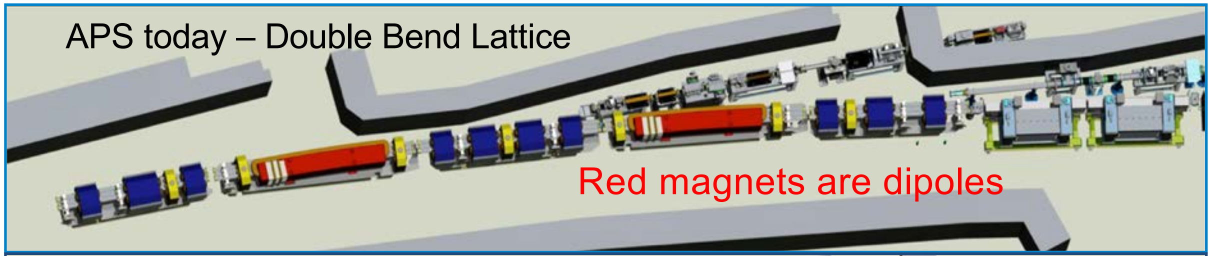
C_L = constant
E = beam energy
N_d = dipoles per sector

APS MBA Upgrade:
Beam energy: 7 GeV => 6 GeV
Dipoles per sector: 2 => 7

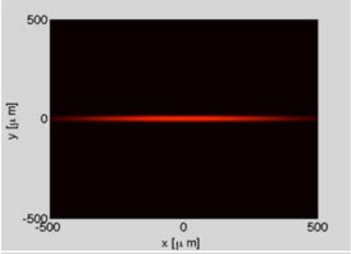
10^2 to10^3 X increase in brightness

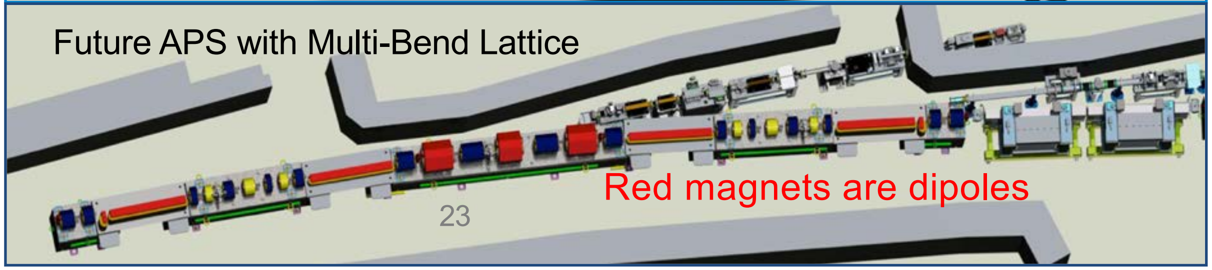


~50-fold reduction in horizontal emittance

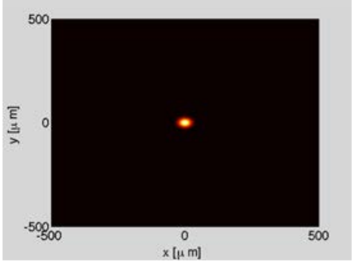

APS today – Double Bend Lattice

Red magnets are dipoles




Future APS with Multi-Bend Lattice

Red magnets are dipoles

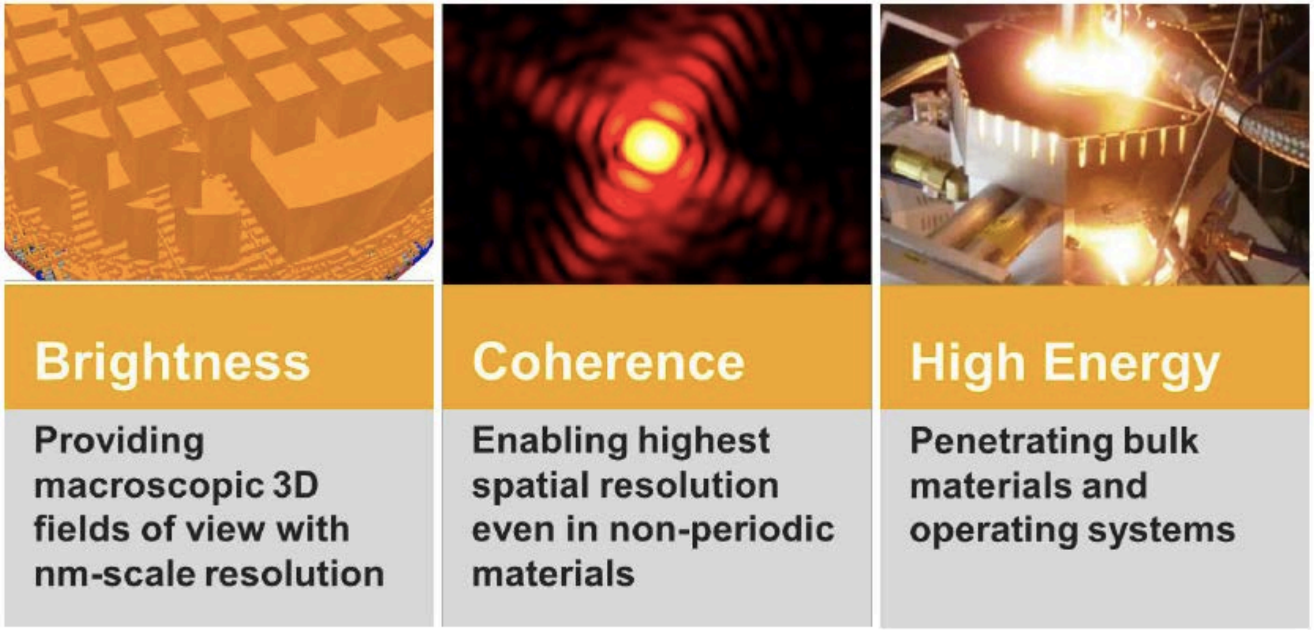


APS-U: A 42 PICOMETER RADIAN SOURCE

- The goal of the upgrade is to reduce the electron beam emittance from the current 3000 pm-rad to 42 pm-rad.

- This will result in a diffraction limited source below 3 keV and much higher coherence at higher energies (about a factor of 100).

APS-U will enable multiscale, 3-D exploration of complex materials and chemical systems – may any atom’s position, identity and motion.



Brightness

Providing macroscopic 3D fields of view with nm-scale resolution

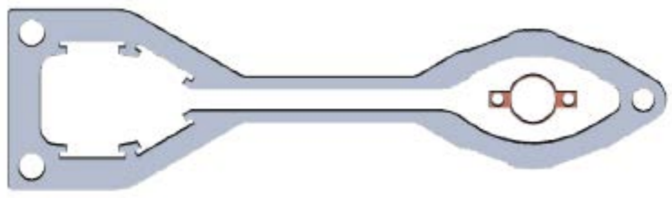
Coherence

Enabling highest spatial resolution even in non-periodic materials

High Energy

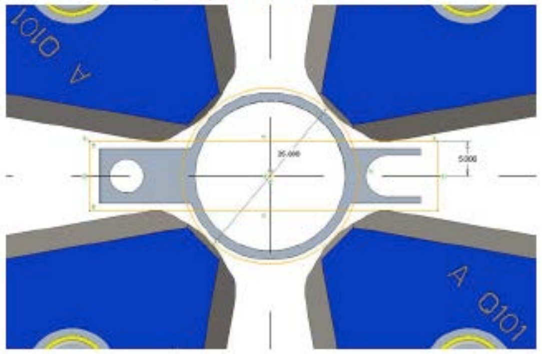
Penetrating bulk materials and operating systems

SMALL VACUUM CHAMBERS AND SCUs



Existing APS storage ring chamber compared to new 22 mm ID APS-U style chamber

The small vacuum chamber permits the focusing magnets pole pieces (blue in fig. on right) to get closer to the electron beam producing higher magnetic focusing fields

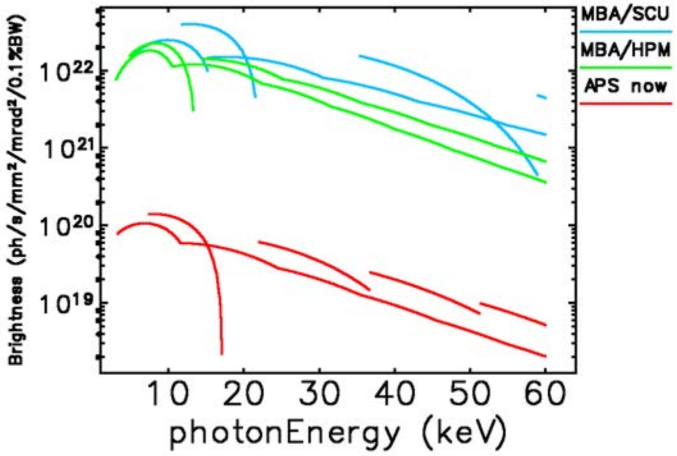


Q1, Q3, Q6 Constraints

- There is a strong desire to maintain (or even increase) x-ray energy of the harmonics from the undulators, how can how do we do this when going from 7 GeV to 6 GeV in the upgrade?
- Go to shorter undulator (ID) periods, i.e., reduce λ_{und}.
- That is difficult to do with permanent magnets as there is less room for the magnetic material between the poles but superconducting undulators (SCUs) can.

SCUs

- 18 mm period
- operate at 4°K
- 0.6 mm NbTi conductor



MBA UPGRADES – A COMPETITIVE LANDSCAPE

ESRF (France)

- ESRF is shut down as of the end of 2018 for an MBA upgrade and will resume operation in 2020.

4 July 2014 Last updated at 06:33 ET

Upgrade for Europe's big X-ray light source

By Jonathan Amos
Science correspondent, BBC News

MAX-IV (Sweden)

- A new (green field site) 3 GeV MBA lattice-based storage ring that is currently operational

SIRIUS (Brazil)

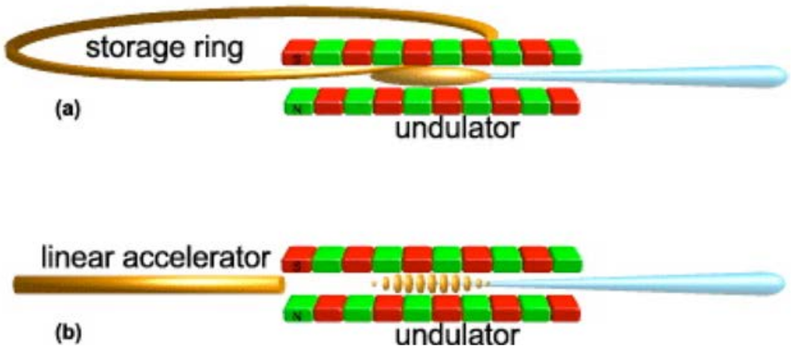
- A new (green field site) 3 GeV MBA lattice-based storage ring beginning commissioning in 2019 (?) and operational in 2020.

SPRING-8 (Japan)

- Planning an MBA upgrading in 2020's

NON-STORAGE RING COHERENT X-RAY SOURCES – X-RAY FREE ELECTRON LASERS (XFELS)

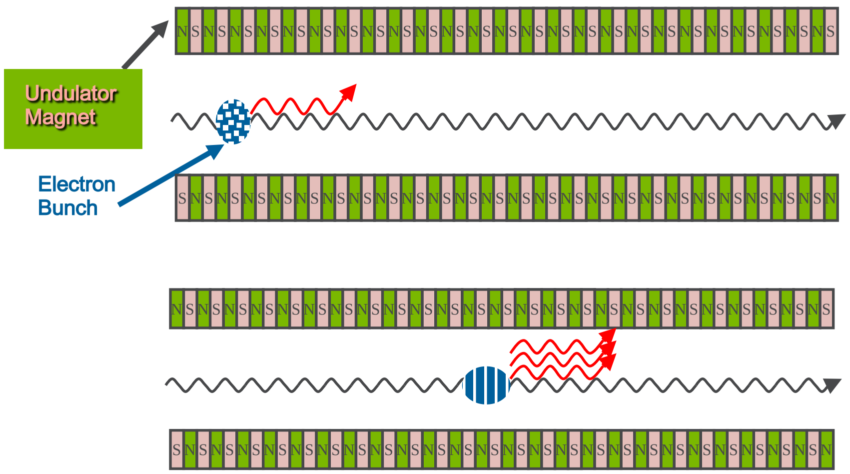
- Another way to reduce the particle beam emittance is through linac-based x-ray free electron lasers (XFELs)
  - Full transverse (spatial) coherence and femtosecond pulses
- An **x-ray FEL** uses the high brightness of an **electron gun** coupled to an **emittance-preserving linac.** You don't have to deal with the equilibrium transverse and longitudinal beam size (length of pulse) that occurs in a storage ring so can have a very low emittance and **very short pulses** (femtoseconds).



- The problem with a linac is you only use the electrons once, and so have to accelerate new electrons for each pulse. The acceleration process is what takes all the electrical power so a different paradigm is required. (In the storage ring you "recycle" the electrons after they have been accelerated up to energy and there is no further increase in particle beam energy as they go around.)

- That new paradigm is to generate "gain" in the electron beam to get more x-rays from a single bunch of electrons - gain is obtained through a process called **Self-Amplified Spontaneous Emission** *or* **SASE.**

SELF-AMPLIFIED SPONTANEOUS EMISSION – SASE



- In this process, an intense and highly collimated electron beam travels through an undulator magnet (≈ 100 meters) producing x-rays.
- If the synchrotron radiation is sufficiently intense (i.e., the undulator is long enough), the electron motion is modified by the E&M fields of its own emitted light.
- Under the influence of both the undulator magnet and its own synchrotron radiation, the electron beam forms micro-bunches, separated by a distance equal to the wavelength of the emitted radiation.

Linac Coherent Light Source (LCLS) at SLAC is an x-ray FEL using 1 km of the 3 km linac there to produce 15 Å to 1.5 Å beams.

John Galayda, LCLS

ENERGY MODULATION → DENSITY MODULATION (MICRO-BUNCHING)

- In this process, an intense and highly collimated electron beam travels through a long undulator magnet (≈ 100 meters).
- If the synchrotron radiation is sufficiently intense (i.e., the undulator is long enough), the electron motion is modified by the E&M fields of its own emitted light.
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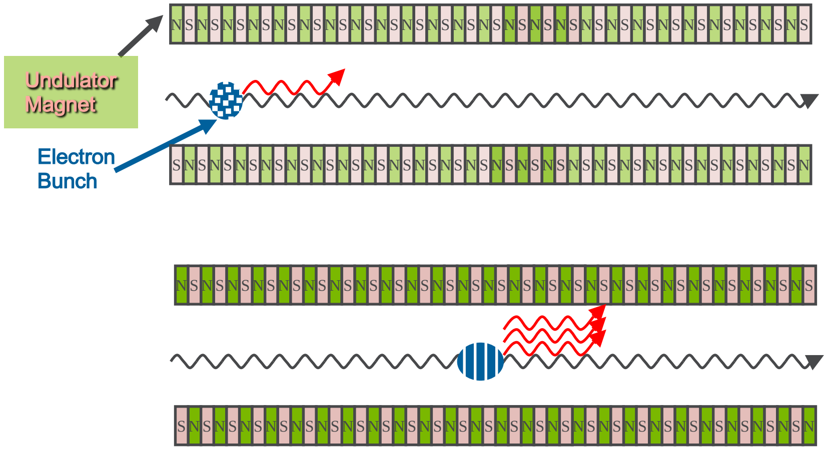


Energy modulation → density modulation

John Galayda, LCLS

EXPONENTIAL GROWTH OF INTENSITY (THIS IS THE ”GAIN” IN THE SYSTEM)

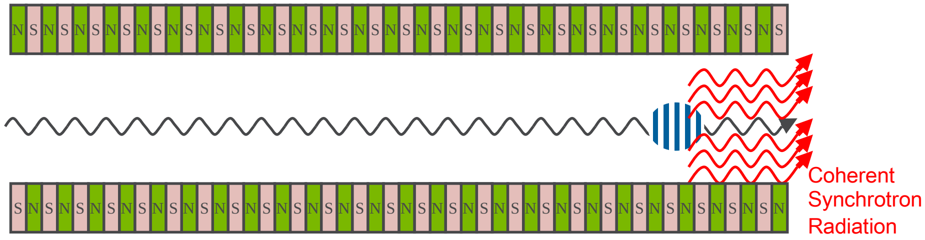
- These micro-bunches begin to radiate as if they were a “single particle” with immense charge – 10^4 the charge of a single electron.

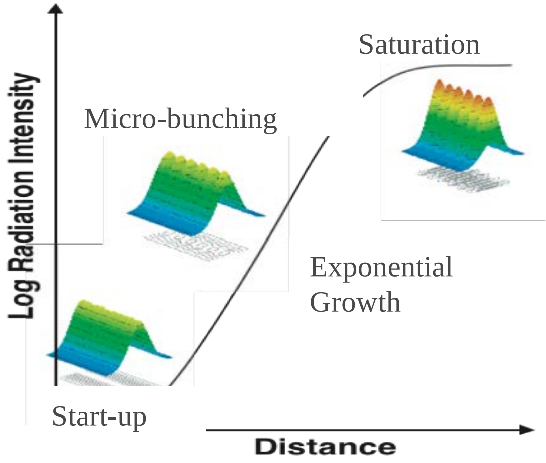
- The lasing starts up from the random micro-bunching (i.e., shot noise) on the electron beam – this is called:

  Self-Amplified Spontaneous Emission (SASE)

- The bunch length in LINACs can be very short resulting in **femtosecond** bursts of x-rays.

See Appendix 5 for more details

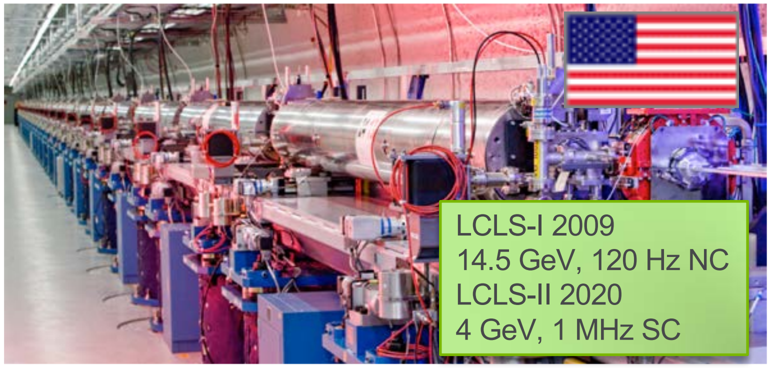




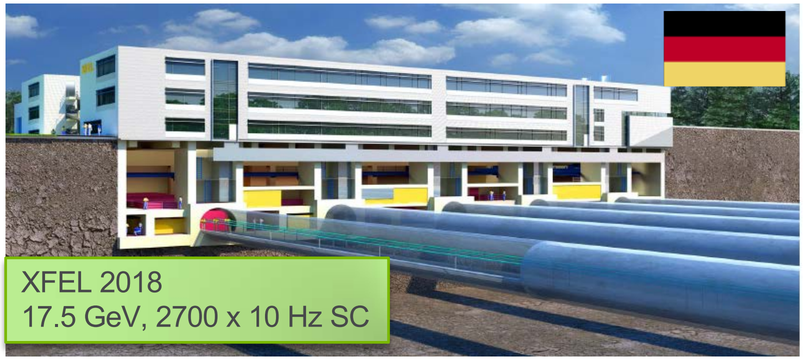
John Galayda, LCLS

- The process reaches *saturation* when the micro-bunching process has gone as far as it can go.

HARD X-RAY FELS IN OPERATION / CONSTRUCTION


LCLS-I 2009
14.5 GeV, 120 Hz NC
LCLS-II 2020
4 GeV, 1 MHz SC


SACLA 2011
8.5 GeV, 60 Hz NC


XFEL 2018
17.5 GeV, 2700 x 10 Hz SC


SWISS FEL 2017
5.8 GeV, 100 Hz NC


PAL XFEL 2015
10 GeV, 100 Hz NC

NC = normal conducting LINAC
SC = superconducting LINAC

SUMMARY

- Synchrotron radiation continues to be an important tool for researchers across all scientific and engineering disciplines.

- There is a strong science case for a new generation of sources such as:
  - Low-emittance storage rings
  - X-ray free electron lasers

- Storage rings can produce partially coherent x-ray beams with megahertz repetition rates but with lower peak power.
  - The APS is working to incorporate a low emittance lattice into the proposed upgrade of the facility (APS-U) that will produce beams of high coherence at megahertz rates.

- FELs can provide very short pulses (femtoseconds) of high peak intensity, coherent x-rays but the repetition rate is currently limited to hundreds of hertz.
  - Superconducting radio frequency linacs will increase that to MHz (LCLS II and the European XFEL).

- Both LCLS II, APS-U, and the upgrade for the Advanced Light Source at LBNL (ALS-U) will keep US x-ray facilities at the cutting edge to produce world class science in the years to come.

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“Handbook of Accelerator Physics and Engineering”, Edited by A. Cho and M. Tigner

QUESTIONS?

APPENDIX 1A: RADIATED POWER FROM CHARGES AT RELATIVISTIC VELOCITIES

The classical formula for the radiated power from an accelerated electron is:

$$P = \frac{2e^2}{3c^3} a^2$$

Where P is the power and α the acceleration. For a circular orbit of radius r, in the non-relativistic case, α is just the centripetal acceleration, v^2/r. In the relativistic case:

$$a = \frac{1}{m_o} \frac{dp}{d\tau} = \frac{1}{m_o} \gamma \frac{d\gamma m_o v}{dt} = \gamma^2 \frac{dv}{dt} = \gamma^2 \frac{v^2}{r}$$

Where $\tau = t/\gamma$ = proper time, $\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$ and $\beta = v/c$

$$P = \frac{2e^2}{3c^3} \frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8}$$

APPENDIX 1B: DEPENDENCE ON MASS AND ENERGY OF RADIATED POWER

$$\boxed{P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}}$$

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e^- with the same total energy, E.)

2. Scales with the 4th power of the particle's energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)

APPENDIX 2: BM SPECTRAL DISTRIBUTION

The spectral/angular distribution of "synchrotron radiation" was worked out by J. Schwinger in 1949. Schwinger found the spectral distribution from an accelerating particle, under the influence of a constant magnetic field, was a smoothly varying function of photon energy and that the spectrum could be parameterized by a critical energy, E_c.

$$E_c = 3hc\gamma^3/4\pi r.$$

Here h is Planck's constant and r the radius of curvature of the trajectory. Note that the critical energy scales as γ^3. In practical units, the critical energy can be written as:

$$E_c[\text{keV}] = 2.218\ E^3[\text{GeV}]/\ \rho[\text{m}] = 0.06651\ B[\text{kG}]\ E^2[\text{GeV}]$$

At the APS the bending magnets have a field strength of 5.99 kilogauss and the ring operates at E = 7 GeV .   The critical energy of the radiation emitted from the BM is:

$$E_c[\text{keV}] = 0.06651\ B[\text{kG}]\ E^2[\text{GeV}]$$

or

$$E_c = 0.06651(5.990)(7^2) = \underline{19.5\text{ keV}}\text{ or }\underline{0.64\text{ Å}}.$$

APPENDIX 3: WHERE DID “K” COME FROM?

Equation of motion for a relativistic charged particle in a magnetic field

$$F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v} \times \vec{B} = ecB_0 \sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

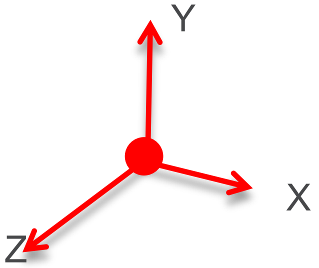
$$\dot{v}_x = \frac{ecB_0}{\gamma m_0} \sin\left(\frac{2\pi z}{\lambda_{ID}}\right) \qquad z = ct$$

$$v_x = -\frac{ecB_0}{\gamma m_0} \frac{\lambda_{ID}}{2\pi c} \cos\left(\frac{2\pi ct}{\lambda_{ID}}\right) = -\frac{eB_0}{\gamma m_0} \frac{\lambda_{ID}}{2\pi} \cos\left(\frac{2\pi ct}{\lambda_{ID}}\right)$$

$$x = \frac{eB_0}{\gamma m_0 c}\left[\frac{\lambda_{ID}}{2\pi}\right]^2 \sin\left(\frac{2\pi ct}{\lambda_{ID}}\right) = \left[\frac{eB_0}{m_0} \frac{\lambda_{ID}}{2\pi c}\right]\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right] \sin\left(\frac{2\pi z}{\lambda_{ID}}\right) = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right] \sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

$$x_{\max} = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right] \qquad \text{and} \qquad \left[\frac{dx}{dz}\right]_{\max} = \frac{K}{\gamma} \qquad \text{where} \qquad K = \left[\frac{eB_0}{2\pi} \frac{\lambda_{ID}}{m_0 c}\right]$$

Magnetic field in y-direction and electron traveling in z-direction



APPENDIX 4: FINITE X-RAY BEAM EMITTANCE

- The Heisenberg Uncertainty Principle sets a lower limit for the product of the size Δx (Δy), and angular divergence Δx’ (Δy’), of radiation. Recall:

primes are used to denote angles

For radiation from relativistic electrons, x’ (y’) is proportional to P$_x$ (small angle approximation).

$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\frac{p_x}{p_z} = x' \text{ or } \frac{\Delta p_x}{p_z} = \Delta x' \text{ and } p_z = \hbar k = \frac{\hbar(2\pi)}{\lambda}$$

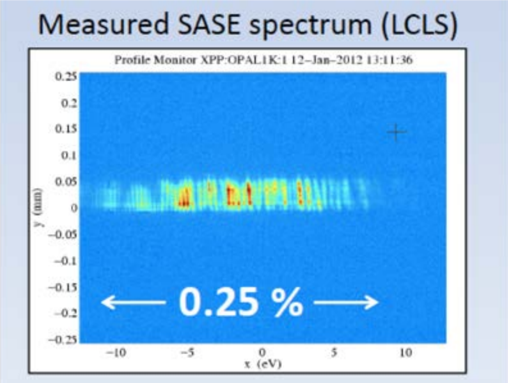
$$so : \Delta x \Delta p_x = \Delta x \Delta x' p_z = \Delta x \Delta x' [\frac{\hbar(2\pi)}{\lambda}] \geq \hbar / 2$$

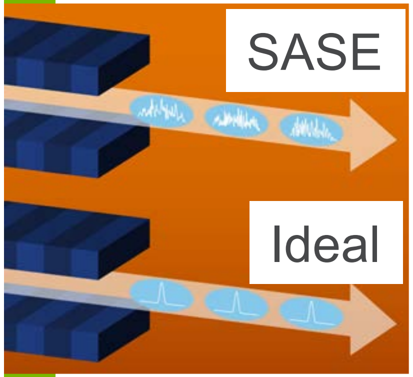
$$\Delta x \Delta x' \geq \lambda / 4\pi$$

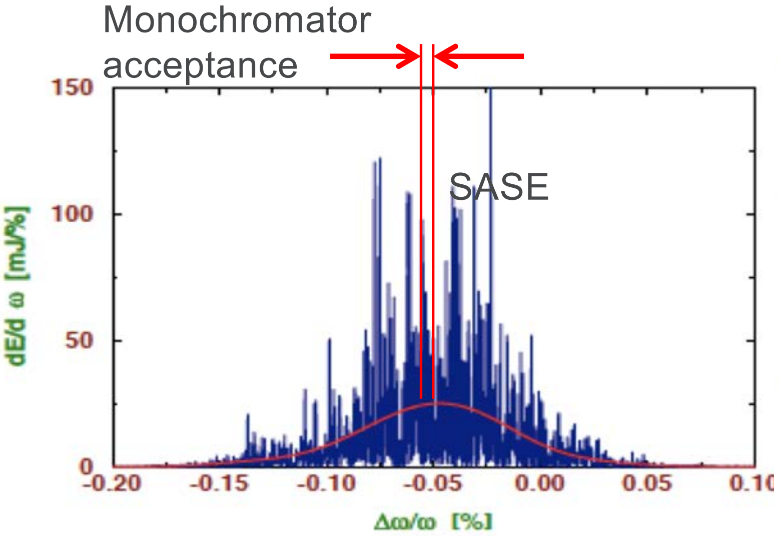
- This says, for a given wavelength λ, the product of its size and divergence cannot be is less than λ/4π. When the product is ≈ λ/4π in both the x and y directions, the radiation is **fully coherent and the source is said to be diffraction limited.**

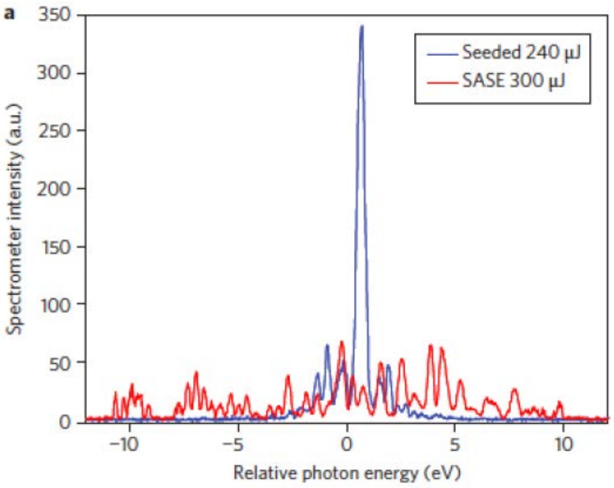
APPENDIX 5: THE SASE PROCESS

- The lasing starts up from the random micro-bunching (i.e., shot noise) on the electron beam instead of being coherently produced by an input “seed” source.


Measured SASE spectrum (LCLS)







One possible “seeding” geometry is to use a diamond crystal to select the wavelength of interest to amplify.